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ORDINARY LEVEL STUDENTS' PERCEIVED ERRORS IN EARTH GEOMETRY: IMPLICATIONS FOR INSTRUCTIONS

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ABSTRACT

This study sought Ordinary Level students' errors in Earth Geometry. It was motivated by the Examination Council of Zambia which pointed out that, the majority of secondary school students are failing to answer Earth Geometry problems. Given a choice, they would not even attempt Earth Geometry questions during examinations. The study was guided by pragmatism, which emphasizes a method that works to find a solution to a problem. A descriptive survey based on a case study of one secondary school and district was carried out. Data was collected from 45 Ordinary Level students from one school and 25 teachers from Mwense district, who responded to a self-reporting questionnaire followed by interviews for in-depth understanding. There was an agreement between teachers and students on the type of errors. Pearson's correlation coefficient r=0.8. The study found that students made errors of calculating the shortest distance between points that are diametrically opposite and those on the same latitude, time difference between places, location of points on the globe, calculating latitude, difference between two places and computation of the circumference of the circle of latitude. Errors were arising from the content of the topic (earth geometry) itself, inadequacy of teachers' content knowledge, teachers' use of lecture method encouraged rote memorization, students' negative attitude toward the topic and the language of instruction. The study recommends district workshop on instructional strategies for Mathematics teachers, introducing a topic language session, structuring content introductory exercises, teaching students using activity based approaches with appropriate teaching and learning aids and integration of Information and Communication Technologies (ICTs) through computer simulations and videos.

Key Words: O-level students, Earth geometry, mathematics instruction.

1. INTRODUCTION
Contextual analysis

Mathematics teaching for conceptual understanding may not be possible if teachers are not aware of the errors that students make in any topic. In fact, the purpose of checking on students' assumed knowledge is to ascertain their levels and anticipate the sort of challenges that they may have. Analysis of students' errors in mathematics forms a strong basis for remedial instruction.

In Zambia, the O-Level Mathematics Syllabus for Grades 10-12 commonly known as Mathematics Syllabus D (4024) consists of a number of topics defined according to six themes namely; Number and Calculations, Algebra, Geometry, Measurement and Estimations,

ISSN: 2582-0745
Vol. 3, No. 02; 2020

Probability and statistics. The topics that fall under the Geometry theme include Geometrical transformation, Mensuration, Similarity and Congruency, Geometrical Construction, Trigonometry and Earth Geometry. This study focuses on students' errors in Earth Geometry as a basis for instruction.

Geometry is a branch of Mathematics that is concerned with spatial visualization. According to Jones (2002), the word 'Geometry' comes from two ancient Greek words, 'Geo', meaning earth and 'Metry', meaning to measure. From these Greek words, one can infer that, geometry is the measurement of the earth. Larson et al (1995) defined geometry as a branch of Mathematics that studies the shapes of objects, their sizes, properties and relationships. In addition, Jones (2002) refers to Geometry as an integral part of people's cultural experiences. It serves as a vital component of numerous aspects of life, from architecture to design in all its manifestations. On the other hand Ali, Bhagawati and Sarmah (2014) assert that Geometry is a unifying theme for the entire Mathematics curriculum and a rich source of visualization for arithmetic algebraic and statistical concepts.

Geometry is closely connected to other branches of Mathematics such as Algebra, Logic, Probability and Trigonometry. To Jones (2002), the teaching of Geometry can as well mean enabling more students to succeed in Mathematics. Geometry can be a topic that captures the interest of learners, especially those who may find other areas of Mathematics, such as number and algebra difficult and abstract. Hoyles and Jones (1998) observe that the learning of Geometry helps learners develop skills such as visualization and critical thinking.

The Examination Council of Zambia (ECZ) examiners' reports for 2015 and 2016, have shown that students' perform poorly in Geometry topics. Such topics have contributed to a high level of overall poor performance in Mathematics. For example, the Examination Council of Zambia (2006) examiners' report showed that questions on Geometry topics such as transformational Geometry were very poorly answered. Furthermore, the Examinations Council Zambia (ECZ) chief examiners' report, (2008) reported that questions on Earth Geometry, Trigonometry, and Transformational geometry were poorly done. Unfortunately, the report did not bring out what could be the learning difficulties, errors made and challenges students face in Earth Geometry.

Akintade (2017: p4) also reports that "report findings over the years indicate that topics such as construction, geometrical proofs, locus, latitude and longitude that prepare students for all engineering courses in tertiary institutions are difficult for many candidates in Nigerian secondary schools." The West African Senior Secondary Certificate Examination (WASSCE) (2010: p237) specifically noted that "latitude and longitude questions were popular among candidates. However, many of them failed to find the latitude of a point. They failed to form the equation, $D = \frac{\theta}{360^0} \times 2\pi R$". One can therefore, deduce that Earth Geometry concepts of latitudes and longitudes are not only problematic among Ordinary Level students in Zambia but also in Nigeria. This problem calls for remedial instructional strategies.

Musonda et al (2018) reports that Earth Geometry as a topic at senior secondary school level has been in the syllabus since 1964 when Zambia got its' political independence and adopted and

ISSN: 2582-0745
Vol. 3, No. 02; 2020

adapted the British Colonial Curriculum. However, in the early 1980s, Earth Geometry was removed from the syllabus but after realizing its significance it was re-introduced in 2004. It is now one of the compulsory topics which does not miss a slot in the Examination Council of Zambia (ECZ) “O” Level Mathematics Paper 4024/1 and optional in Mathematics Paper 4024/2. In section B of Mathematics Paper 4024/2, when it comes as a stand-alone question, the question carries a total of 12 marks and when it comes as a part question, it carries 6 marks.

Currently (2019), Earth Geometry is a topic taught at Grade 12 level. It deals with location of places, calculation of distances between two points and determination of time differences between points on the Earth’s surface. According to the Curriculum Development Centre of Zambia (2013: p27), the Ordinary Level Mathematics Syllabus for Grades 10-12 commonly known as Mathematics Syllabus D specifies that students be taught the following content in Earth Geometry:

- Introduction to Earth Geometry, Small and Great circles, Latitudes and Longitudes
- Distance along Latitudes and Longitudes and Speed in Knots and Time variations

One notable thing on the content is the sequence or orderliness of the topics. This entails that Mathematics teachers can follow the stipulated order if students are to learn and understand geometry concepts effectively. Failure to follow the order may result in students facing challenges in understanding the concepts and ultimately make errors.

Regarding the learning outcomes, Curriculum Development Centre (2013: p27) specifies the following as expected outcomes of teaching Earth Geometry:

1. Explain the concept of Earth Geometry
2. Distinguish between Small and Great circles
3. Calculate distances along parallels of Latitudes and Longitude in kilometres and nautical miles. (The use of miles although in Zambia we use km, is a reflection of colonial education and context)
4. Calculate the shortest distance between two points on the surface of the earth
5. Calculate speed in knots and time variations

Failure to achieve these learning outcomes is shown by students making errors during class exercises, tests, mock examinations and final examinations. Consequently, Mathematics teachers are called upon to ensure that they are acquainted with students’ errors as they make schemes of work and lesson plans. It is worth noting that the above outlined Earth Geometry content and expected outcomes are the ones students are failing to answer adequately during final examinations.

Statement of the Research Problem

ISSN: 2582-0745
Vol. 3, No. 02; 2020

O-level students in Zambia are making errors in Earth Geometry resulting in them underperforming in mathematics at Grades 10-12. Successive chief examiners reports produced by the Examination Council of Zambia such as ECZ (2004) and ECZ (2006) have cited Earth Geometry as a problematic topic. Ministry Of General Education (MOGE) in Zambia has introduced interventions such as In-service Education Training (INSET) and also involved International Corporation in strengthening Science, Mathematics and Technology Education (SMASTE). However, despite all these efforts, the problem of students performing poorly in Mathematics and in particular Earth Geometry has continued to haunt the Ministry of General Education in Zambia. There is limited literature on what errors these students make. Academic researchers have given limited attention to rural secondary school students' views on challenges they face in learning Earth Geometry. This study contributes students' errors as a basis for remedial instructions to reduce students' challenges in Earth Geometry in Zambia.

Research Questions

This study sought answers to the following pertinent questions:

(1) What are secondary school students' errors when answering Earth Geometry questions?

(2) What factors account for students' errors when answering Earth Geometry questions?

(3) What instructional strategies can reduce students' errors in Earth Geometry?

Research Objectives

The study intends to:

(1) Identify secondary school students' errors in answering Earth Geometry questions.

(2) Deduce factors accounting for students' errors in answering Earth Geometry questions.

(3) Suggest instructional measures to reduce students' errors on Earth Geometry questions.

Significance of the study

The main purpose of this study is to improve the teaching of Earth Geometry in secondary schools in Zambia. Identification of students' errors is a critical basis for remedial instruction. Data gathering exercise will raise awareness among teachers of the need to carry out error analysis on earth geometry. It is hoped that the information obtained from the study can be used by stakeholders in the Ministry Of General Education in Zambia as a rich source of mathematics teachers' staff development programs. To that end, the study is a training needs identification exercise. Ultimately the study contributes literature on the teaching of Earth Geometry.

2. LITERATURE REVIEW

Studies on Students' Errors in Geometry Learning

ISSN: 2582-0745
Vol. 3, No. 02; 2020

According to Musonda et al (2018) the debate surrounding the teaching of Geometry was complicated in the middle of the nineteenth century as a result of moving the course from college level to high school level. Although the maturity level of the learners at secondary school is lower compared to college level students, there was no organized adjustment in course contents (Clements, 2001). The omission entail that the same content taught at college level was moved down to secondary level. Although one can teach any content at any level, this study wonders whether all teachers in Zambia are able to dilute Earth geometry content to the appropriate levels of their students.

Although changes were called for, William (1968) cited by Musonda et al (2018) revealed that Euclid's traditional approach to the subject is considered by many to be a significant part of man's cultural heritage, hence had to be maintained. Unfortunately, what Williams (ibid) called "man's cultural heritage" is not the same cultural heritage in Zambia due to ecological differences. Usiskin (1983) and Hoffer (1986) cited by Musonda et al (2018, p32) noted that, many students experienced difficulty writing proofs and most of them were unsuccessful in solving geometrical problems in the traditional Euclidean geometry. In the Zambian Ordinary Level Mathematics Syllabus, students are not required to show proofs when answering Earth Geometry questions. However, students are required to solve Earth Geometry problems using appropriate formulae.

According to Clements and Battista (1990), the underlying concept in geometric thought is spatial reasoning which is the ability to see, inspect and reflect on spatial objects, images, relationships and transformations. As such the teacher is expected to ensure that his or her students visualize figures, shapes and planes that may not be very obvious to the student. Clements and Battista (1990) suggest that students make errors due to low spatial reasoning which is unique and difficult to teach and learn without models. Therefore, this study intends to establish remedial methods for teaching and learning of the spatial reasoning part of geometry and Earth Geometry in particular.

Battista (2007) identified students' geometric conception as the main error. It affects students' perception. What one sees is affected by what one knows and conceives. This calls for teachers to link new geometric images to what the student knows. For example, a sphere can be linked to a ball or orange. Battista (ibid) noted that such errors are enhanced by teacher drawn diagrams with errors as data or representations. To buttress the point, Chazan and Yerushlmy (1998: p70) said "diagrams are aids for intuition and are not necessarily the objects of study themselves." There is need therefore, for this study to establish the nature of diagrams that teachers can use to enhance students' spatial understanding of geometry.

One of the factors which made Geometry learning difficult was the Geometry language which involved specific terminology unique to the African child's vocabulary (Ojose, 2011). It needs particular attention and understanding before it could be used meaningfully. Further, Bishop (1986) cited by Tembo (2013) and Lappan (1999) suggested that, the misuse of Geometry terminology could lead to misconceptions of geometric knowledge. These commends motivates this study to establish the influence of language in students errors in earth geometry.

ISSN: 2582-0745
Vol. 3, No. 02; 2020

Commenting more on the importance of Mathematics language in Africa, Macforlone (1990) argued that, Mathematics conceptual development for African students is affected by a lack of a curricular and teaching material specifically designed for Africa. Chinamasa, Nhamburo and Sithole (2014) also concluded that Mathematics textbooks and instruction lack local conceptualization. Foreign conceptions are promoted by African teachers in African countries using English as a medium of instruction to teach African learners. For example, in Zambia, English is the only official medium of instruction from Grades 5 to tertiary and higher learning institutions. However, the language of instruction policy contradicts the suggested teaching and learning method in the Ordinary Level Mathematics Syllabus. The syllabus encourages teachers to expose learners to practical work as much as necessary through contextual reference to the local environment. The local environment mentioned includes the use of local language in Mathematics teaching generally and Earth Geometry specifically. Thus, this study will get views from students on whether there is a possibility of using familiar local languages in Earth Geometry teaching and learning.

Chinamasa (2014) cites Jaji (1992) who postulates that English as a medium of instruction has an inhibiting factor. Jaji (1992) established that Form 2 pupils in Zimbabwe lack Mathematics reading skills and language. This is a critical factor for Earth Geometry questions which often are presented in word forms. Nziramasanga's (1999) report concurs with Jaji (1992) and suggested that Mathematics teachers should use the mother language to develop pupils' mathematical concepts as a strategy for reducing conception errors. The critical issue in this case is the language used by teachers to develop understanding of Earth Geometry in their teaching. Therefore, this study suggested a language teaching lesson for Earth Geometry which can reduce language blocks.

According to Chakerian (1972), many teachers have used lecture, group discussion and question and answer methods to teach Earth Geometry. These methods are popular among teachers of Mathematics in Zambia. Lecture method is popular for covering more content within shortest possible time and is applicable to overcrowded classrooms. However, the lecture method should not solely be relied upon in the teaching of Earth Geometry since it promotes rote learning on the expense of spatial understanding. On the other hand, group discussion method promotes team work. Therefore, group discussion can be used with caution for Earth Geometry to avoid students sharing misconceptions exhibited as errors in examinations.

According to Hoyles and Jones (1998) teachers tendency to teach Geometry by informing learners of the properties associated with planes or solid shapes, and then completing the exercises contributes to poor performance in geometry. Lecturing has limited attempts to encourage learners' thinking and reasoning skills. Hoyles and Jones (1998) suggest that students' poor performance in Geometry topics like Earth Geometry is due to teacher centered approaches such as lecturing method. Therefore, this study suggested teaching methods which involve students.

Kalejaiye (2000) also contends that poor performance in Geometry is a result of teachers not involving learners in their teaching and the adoption of the rote learning style. Teaching by informing students' about geometric terminologies does not link class work and real-life

ISSN: 2582-0745
Vol. 3, No. 02; 2020

situations. In fact students will not see its relevance. Thus, there is need for Mathematics teachers to involve students in the teaching of Earth Geometry to reduce conception errors.

According to Mullis (2000), in many Geometry classrooms today, teachers merely introduced learners to facts about Geometry and then drilled them with concepts in deductive reasoning. Students' were seldom given the opportunity to discover and conceptualize geometry on their own. Additionally, Hoyles and Jones (1998) argue that although the deductive method is central to Mathematics teaching, providing a meaningful experience for learners at school appeared to be difficult. Such observations suggest that teaching methods contributed to students' poor performance in Geometry and Earth Geometry in specificity.

Geddes and Fortunato (1993) emphasized that quality of instruction was the greatest factor influencing learners' acquisition of Geometry knowledge. Unfortunately, they did not stipulate the indicators of quality. To that end, Strutchens (2001) cited by Musonda et al (2018: p67) advised teachers to put emphasis on hands-on explorations, developing geometric thinking and reasoning, making conjectures and carrying out geometric projects.

Tembo (2013) revealed that both teachers and learners had challenges in teaching and learning Earth Geometry. Teachers' challenges ranged from inadequate knowledge of the topic to lack of resources in order to teach effectively. In addition, teachers did not get adequate support in the area of Earth Geometry in their teacher preparation programs. As a result, they went into the teaching field with the same conception errors that they had when they were pupils themselves in school. Further, the study revealed that teachers found it hard to explain or introduce some concepts in Earth Geometry to secondary students. Specifically calculation of the distances along latitudes and longitudes and explaining angles of latitude and longitude was a challenge. Calculation of the shortest distance via the poles was also difficult. The study also showed that students' faced the challenges of comprehending the language or geometry terms. Students blamed their teachers for poor teaching. One wonders whether the students know what is good and poor teaching.

Further, the study by Tembo (2013) also revealed that the majority of teachers used the lecture and discussion approach in their instruction in Earth Geometry. Tembo (ibid) suggests non-routine and hands-on activities to enhance geometry thinking. Such student centered methods can provide opportunities for students' to discover and explore on their own.

Simukoko and Sakala (2018) carried out a study to investigate the impact of Earth Model in understanding of Earth Geometry by in-service Student Teachers at Mukuba University. The study indicated a statistically significant difference in the post-test scores for the experimental group (Mean = 60, standard deviation = 19.28) and the control group (Mean = 42.36, standard deviation = 17.98), $p = .01$. Therefore, Simukoko and Sakala (2018) concluded that incorporating Earth Model in teaching Earth Geometry has a positive impact on in-service student teachers understanding of Earth Geometry. Further, the study also revealed that students had challenges in calculating the surface area between two meridians and the shortest distance between points on the same latitude which are not diametrically opposite. Additionally, the study

ISSN: 2582-0745
Vol. 3, No. 02; 2020

suggested that the concepts of Geometry are abstract and require more visualization tools to aid students' understanding.

Findings from teachers cannot be generalized to school children because of age difference and locality. Hence, conducting the current study is fundamental because it will show the influence of Earth Geometry Models as a teaching method from students as participants. Lappan (1999)'s recommendation for more visual activities in the classroom also suggests the use of models.

According to Bishop (1983) being able to "touch-see-do'' and interact with the objects of their learning promotes students' learning of geometry in a more imaginative and successful way. In the Theory of Multiple Intelligences, Gardner (1996) cited by Tembo (2013) suggests that kinesthetically inclined students learn best when actively involved with the objects on their learning. In this case, geometrical concepts require visual interpretations since many geometry problems are presented in a two-dimensional format.

3. METHODOLOGY

Research Design

The study is guided by the Pragmatism research philosophy which priorities what-ever method works to solve the problem. A case study descriptive survey of secondary school students' perceptions of errors made in Earth Geometry, factors accounting for the errors and instructional methods to reduce the identified errors was done. Chikoko and Mhloyi (1995) cited by Maregedze, Chinamasa and Hlenga (2012: 148) support this by defining a descriptive survey as "a method of research which describes what we see." Mustafa (2010) cited by Chinamasa, Nhamburo and Sithole (2014: 62) also contend that the major purpose of descriptive survey is to describe the state of affairs as it exists. The researcher will not manipulate variables. Further, descriptive surveys also use different methods of data collection to enhance method and data source triangulation. In this study it will enable the researcher to administer questionnaires, conduct face-to-face semi structured interviews with secondary school students and carry out document analysis. Surveys can be extremely efficient at providing large amounts of data at relatively low cost in a short period of time.

Population and sampling

The population of this study was composed of all teachers and students registered for School Certificate Examinations (SCE) for 2019 in Mwense District. All the students had learnt and teachers had taught earth geometry. They were rich sources for the errors encountered in learning Earth Geometry. Their teaching and learning experiences are critical for this study. Since the purpose of the study is to understand and find a solution, a purposive sample of one school was adequate. It had 45 students, so the study sample size, n =45. This sample size (n = 45) is statistically a large sample for the variable to be normally distributed and findings generalized to similar schools. A purposive sample of 25 teachers from Mwanza district who were at a marking session participated in the study. Coyne (1997) opines that the logic and power of purposive sampling lies in selecting information rich cases for the study in depth. This is further supported by Patton (1990) who contends that purposively selected informants are preferred for the reason

ISSN: 2582-0745
Vol. 3, No. 02; 2020

that they are likely to be more conversant or well-informed about the phenomenon the researcher is investigating. In this case, the participant inclusion criterion was: being available, having the variable (errors on earth geometry) and willing to participate.

Instruments

The study used three instruments to collect the data; a teachers' and students' questionnaire, interview guide and students' mathematics note and exercise-books and teachers' mathematics schemes of work.

A questionnaire was the main instrument considered ideal since all the respondents are literate, able to understand the questions and express their views in writing. The study required individual perceptions collected within a short period of time. The questionnaire collected respondent demographic data and perceptions on errors in Earth Geometry. Students were also asked to identify errors, their possible sources and how they could be reduced. The other instruments used are semi-structured interview guides.

The interview guides, required participants to explain how they got certain incorrect answers identified in their exercise books. The interview captured students' inner thoughts and possible conception error sources. The student's exercise –book showed what the students were exposed to during the lesson. Teachers' mathematics schemes showed what objectives they intended to achieve in each lesson. These were ideal for the interaction between the teacher and students.

Data collection and Analysis

Researchers sought permission from the District Education Board Secretary's (DEBS) office in Mwense district and secondary school head teachers to gather data for the study from their schools. This was an important stage to observe ethical issues. A pilot study was done at one secondary school not participating in this study. Twenty five teachers from secondary schools in Mwense district, also responded to a questionnaire which sought students' errors and possible teaching methods. Seven teachers' mathematics schemes were analyzed for earth geometry objectives and time. Assistance from colleagues who taught at sampled secondary schools was sought for hand delivery of questionnaires to the respondents in those schools. Researchers analyzed teachers' mathematics schemes of work for objectives on earth geometry. This was followed by analysis of students' marked test exercises for errors. Interviews were carried out with students and teachers to establish sources of errors and confirm what was reported on questionnaires.

Data analysis started by screening questionnaires for completeness and answering key questions. Open ended responses were captured on frequency tables. Percentages were used to compare distributions of responses. Pearson's correlation coefficient was calculated to check for the level of agreement between teachers and students. Emerging themes on errors, possible sources and teaching strategies were raised. Further, qualitative answers were presented verbatim to maintain originality.

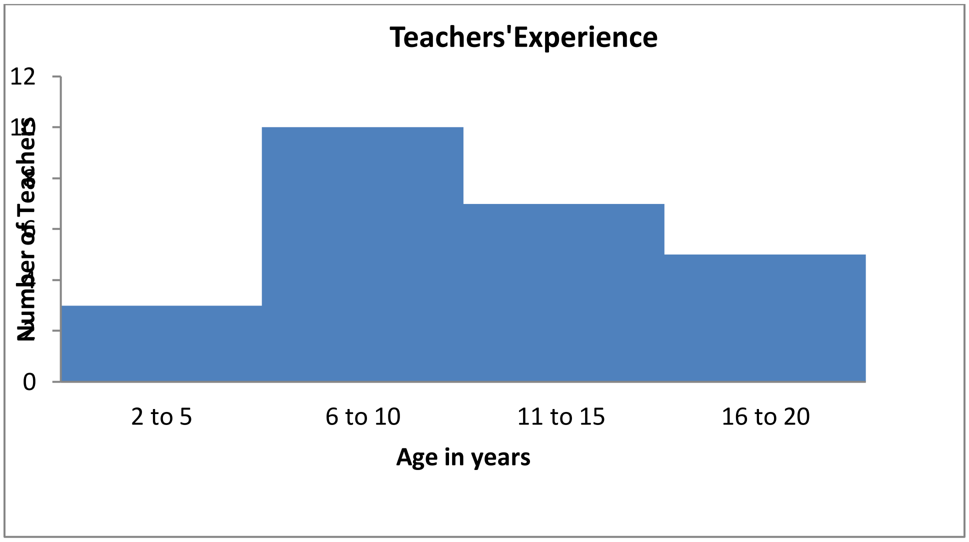
ISSN: 2582-0745
Vol. 3, No. 02; 2020

4. FINDINGS AND DISCUSSIONS

Response Rate:

Data presented in this study was derived from 25 teachers, 7 mathematics schemes of work, 45 O-level students and 9 students' mathematics note books and exercise books. These were considered valid sources for students' errors in earth geometry and their possible sources.

Teachers' Experiences **n=25**



The majority of teachers had been teaching mathematics for ten years. They are experienced enough for their responses to be relied upon. They are rich sources for students' errors and possible teaching strategies which are ecologically viable.

Teachers' Age Distribution **n = 25**

Stem	Leaf
2	5 6 6 6 7 8 8 9 9 9 9
3	0 1 1 3 5 6 7 7 9
4	2 4 6 9
5	
6	2

Key: 4 |3 = 43

The majority of teachers who participated in the study are young, 25 to 29 years. These are still receptive to any new teaching methods. The mode =29 < Median = 30 < Mean = 34.14. The age distribution is positively skewed. There is one outlier old teacher aged 62 years.

Students' Computational Errors N= 70

Type of Computational error	Teachers (n=25)	Students (n=45)	Total N=70
Distance between diametrically opposite points	17	19	36 (51%)
Distance between points on the same latitude	9	10	19(27%)
Time between places located by longitudes or latitudes	14	32	46(66%)
Location of point on the globe	15	28	43(61%)
Circumference of circle latitudes	8	17	25(36%)
Application situations	20	41	61(87%)

The table shows the number of participants suggesting each error. The majority of teachers (20) and students (41), reported that students failed to identify application for Earth Geometry content. A correlation analysis for errors identified by teachers' and students' had Pearson's Correlation Coefficient, r = 0.8. This shows that, there is a strong positive correlation between teachers and students identification of errors. In fact the coefficient of determination r^2 = 64%, implies that, 64% of students' errors are dependent on teachers. In this case teachers are the independent and students dependent variables. These findings support Musonda et al (2018) who found that there was no section of earth geometry that was easy for all participants.

Document analysis of teachers' schemes showed that they had simply copied objectives as presented in the Curriculum Development Centre (2013, p27). Their evaluations did not reflect students' actual attainment of objectives. For example, the average mark for a test on calculating shortest distances between latitudes was 32%. The teacher's evaluation was: "*The lesson was successful. Students actively participated in the lesson*"

We concluded that, teachers need staff development on measurement and evaluation specifically on test error analysis. Guidance from the use of objectives model for evaluation is critical.

During oral interviews, students indicated that calculating shortest distance was challenging. This finding was in agreement with one of Musonda et al (2018) findings which revealed that, calculating shortest distance between two places on the same latitude was one of the areas that was endorsed by the majority of both teachers and learners.

Student D said, "W*hen I see the question on shortest distance, I don't even waste time. I just skip it and answer other questions. Shortest distance is very challenging and confusing.*"

Student U, added, "We *hear from former pupils that the topic is very hard. So we do not have interest in the topic. In fact, we just pray that the teacher does not teach us earth geometry.*"

This was a student's defeatist stereotyped position. Interview probing questions revealed that students do not even distinguish between calculating shortest distance between two places that are diametrically opposite and non-diametrically opposite (lie on same latitude). Students' pigeonholed stance affected their will to attempt earth geometry questions.

Student X, had the following to say, "*I have never heard of diametrically opposite and non-diametrically opposite points. This is very new to me sir. Are such things there?"*

We deduced that, the language 'diametrically' was not understood or linked to the word 'diameter'. We deduced the need for teachers to start concepts from the very basics. Models and illustrative diagrams can illustrate these. A language lesson for each chapter is called for.

The use of familiar language was mentioned as the least remedy in the reduction of perceived challenges in earth geometry. This is because according to Jaji (1992) the medium of instruction, English is an inhibiting factor. This is a critical factor for earth geometry problems which are too presented in word forms like the case of linear programming

Factors contributing to Students' Errors in Earth Geometry n = 70

Factors related to the subject	Factors related to Teachers and Teaching	Factors related to learners
-too many complex formulae to remember -earth geometry is difficult -long process of answering the question -language terms are foreign and difficult to comprehend -topic is too long	-teachers do not know the subject content -teachers teach earth geometry hurriedly -lack of appropriate teaching/ learning aids -lack of local and practical examples -use of lecture methods -use of abusive language	-lack of motivation for geometry -absenteeism leading to missing concept -lack of self- study habits -fear of the subject and topic

Analysis of the contributing factors indicates that the factors revolve around the content of earth geometry itself, teachers' content and pedagogical knowledge and students themselves. The teacher is between the subject content and students, hence a critical variable activating all other variables. This finding suggests that, developing teachers' content and teaching methods can reduce students' errors in mathematics and earth geometry in particular.

During the interviews, student P said, "O*ur teacher is too fast when teaching the topic. When we ask, the teacher gets annoyed, furious and use abusive language on us".*

Student M, echoed the time factor by saying *"more time must be allocated when teaching the topic. Not just one week like our teacher did. There is need to be teaching slowly. I propose one concept per contact."*

ISSN: 2582-0745
Vol. 3, No. 02; 2020

Teacher X, defended the speed in these words, "*Gentlemen, we teach students to pass National Exams. The syllabus must be completed, otherwise students will fail.*" This seemed to suggest that, a centralized school curriculum is not ideal for students' content understanding.

Student D, had the following to say, "O*ur teacher just came once with the earth model which was even confusing. The rest of our lessons were just theoretical. The teacher was just writing diagrams on the board and then solves examples with us.*"

Finding of this study on poor teacher pedagogy was similar to that of Musonda et al (2018) who found that students' difficulty in understanding earth geometry was attributed to teachers' failure to explain concepts and inspire learners. Further, lack of teaching aids made it difficult for pupils to visualize the spherical nature of earth in three dimensions. Clements (2001) contends that it has been established that when learners failed to grasp the concepts, they resorted to memorization and complain that the topic had too many formulas to memorize.

During interviews, Student T, had the following; "T*he problem is because some students do not come to school every day. You will find a pupil coming to school when the topic is half way in completion. So the problem is within us*".

The revelations by the students clearly show that students also have a problem. Musonda et al (2018) also found that the most cited reason for learner's poor grasp of the topic was attributed to learners' negative attitude towards mathematics which contributed to irregular attendance.

Study implications for Instructions

We considered educational instruction as a process, starting from consideration of the goals of teaching earth geometry, resources, lesson delivery methods and ending with student and teacher self- evaluations. This study found that, students' calculation errors are indicators of the objectives of Curriculum Development Centre (2013, p27) not being achieved. Factors contributing to the errors are in three: the topic, earth geometry which is not well known by teachers, the teachers who are failing to teach it using practical examples and the students who are not motivated by the teachers' teaching methods.

In view of these, this study recommends the following measures to reduce students' errors in earth geometry.

Mwense district mathematics education officers can organize a mathematics teachers' workshop on the teaching of Earth Geometry. Facilitators can be found to facilitate the following workshop content:

a) Teach the content for Earth Geometry. This is important for old teachers who find it for the first time as material to teach. Facilitators can provide handouts on this content.
b) Discussions of practical examples and applications of earth geometry.
c) Writing of text-book chapter for earth geometry to include relevant examples.
d) Making of physical (wire) models or computer assimilations of the cross-section of the globe. These can be used to show longitudes and latitudes.

e) Holding demonstration lessons on how the models can be used for instruction in class.
f) Use of project work for earth geometry in class.
g) Setting earth geometry assignments with method marks being awarded for analysis.
h) Structuring of marking guides for earth geometry projects, assignments and classwork.
i) Students' error analysis as a teaching learning tool

At school level, the study recommends these:

a) Allocation of at least 3 weeks for teaching earth geometry. The suggested content for each lesson plan is as follows:
Week 1; explanation of the concept of earth geometry and its significance, distinguish between small and great circles, location of points to the labeled sketch of earth, labeling of points on the sketch of the earth, circumference of the earth including great circles, radius of small circles, circumference of small circles.
Week 2; conversion of distance from kilometers to nautical miles and vice versa, longitude difference between two places, latitude difference between two places, calculate distance along longitudes in kilometers and nautical miles, calculate distance along parallel circles of latitudes in kilometers and nautical miles.
Week 3; calculate shortest distance to and between two diametrically opposite points, calculate shortest distance between two places that lie in the same latitude, calculate speed in nautical miles, conversion from degrees to hours and vice versa, time calculation and earth geometry applications.
b) School authorities can procure teaching aids such as earth models. Teachers can improvise teaching aids for effective teaching of earth geometry. An example of an improvised teaching and learning aid can be an orange. The teacher can use an orange when teaching earth geometry concepts such as shape of the earth, longitudes and latitudes. The teacher might say to students, "this is an orange representing the earth. Draw it in your books." The teacher can then guide students to cut an orange horizontally and vertically. The horizontal cutting of an orange will illustrate parallel circles of latitudes while the vertical cutting will illustrate longitudes which run from North Pole to the South Pole.
c) Recap pre-requisite concepts using structured introductory exercises. For example, before teaching the concept of location of points, circumference of great circles, distance between two points on the same longitude and radius of latitudes, let teachers give introductory exercises on coordinate geometry, mensuration (circumference of the circle and arc length calculation) and trigonometry respectively.
d) Teachers to help students realize the importance of earth geometry in various fields such as navigation. Teachers can do this by making students watch a video showing an aero-plane or navy ship getting lost because of the pilot or ship captain not knowing the place. The teacher can also illustrate the importance of earth geometry by getting extracts from the national budgets showing the money allocated to airport upgrades or training of marine soldiers. Guest lecturers can be invited from suitable fields.
e) Teachers to involve students when making improvised teaching aids on earth geometry such as earth models using locally materials from students' homes. Some of the examples

of local material aids can be an orange, traditional made ball using balloons with threads from motor vehicles as well as earth model made from disposed wires.

f) Provide more problems for students to practice solving and marking. This study found that earth geometry was covered on average four lesson contacts and only four problems were done by students. We recommend that earth geometry be covered in at least 18 contacts and problems respectively. We further recommend an end of topic test given to students immediately upon completion of the topic.

g) Teachers are also encouraged to learn how to download audio lesson videos on the internet and save them on computers. This is because there are a lot of videos on the internet that once integrated in the teaching can result in effective learning. Videos show how certain earth geometry concepts can be delivered using simple and local materials. Such videos can also be shown to students. Further, the GeoGebra software can be easily downloaded on (http://wwww.geogebra.org) and used in earth geometry teaching. This software can enable the user (teachers and students) to coordinate various representations of a mathematical idea in a dynamic way and further gain insight into the focal mathematical structure.

h) More continuing professional development meetings in form of Lesson Studies to be encouraged in schools. Let those teachers conversant with the topic help the other members in the department. If in the mathematics department there is none conversant with the topic, we recommend that an external conversant teacher be hired to help the department in the topic.

i) Teachers to be deriving earth geometry formulae together with students. Let teachers be coming up with activities that enables students derive earth geometry formulae. The habit by many teachers of just writing formulae on the board and tell students to master them should not be encouraged.

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