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APPLICATION OF THE RHODESCRIPT THEORY TO GEOMETRIC CONCEPTS IN THE 5TH GRADE OF PRIMARY SCHOOL

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ABSTRACT

According to global assessment indicators, such as PISA and TIMSS, the performance of Greek students (Hellenic and Cypriot) in Mathematics is below the average of the countries participating in these assessments. Given these evaluations, this research presents the results from the application of RhodeScript Theory in the teaching of mathematics in Elementary School, with the aim of strengthening and improving mathematical literacy. In particular, this paper examines the difficulties faced by 5th grade students in Geometry subjects and seeks to reduce the cognitive barriers that appear in these concepts, according to the relevant literature, through teaching interventions using the mathematical tools of RhodeScript Theory. The research findings showed that students who participated in activities in which RhodeScript Theory was applied scored high on assessment projects and developed a positive attitude towards geometry. In addition, students abandoned their misconception that a triangle is the shape that has equal sides and angles.

Keywords: RhodeScript Theory, Teaching interventions, 5th grade of elementary school, Geometry.

1. INTRODUCTION

The declining course of mathematical school education for the Greek educational system is evident from the results and performances of Greek students in international programs for evaluating countries' educational systems, such as the PISA program (Programme for International Student Assessment), and the Organization for Economic Co-operation and Development (OECD). In this international assessment program, Greece is consistently below the average of participating OESD countries, with the 2012 and 2015 competitions ranking our country 42nd and 43rd, respectively, in mathematics out of a total of 72 countries (PISA, 2003, 2016).

It is not only our country that faces the above problems, but others similarly, which explains why in the international literature review there is a multitude of research that have dealt with the difficulties faced by students in mathematics. Specifically, the research findings show the critical aspects of how mathematics is taught (Chen & Li, 2009; Rønning, 2013; Howe et al., 2015; Vlachou & Avgerinos, 2018; Psaras et al., 2020, Vlachou et al., 2020), the use of representations during the teaching of mathematical concepts (Shahbari & Peled, 2015; Dreher & Kuntze, 2015; Vlachou & Avgerinos, 2019) the attitudes and beliefs of teachers and future teachers towards mathematics (Avgerinos & Vlachou, 2013; Şahin, Gökkurt & Soylu, 2016; Thanheiser et al., 2016; Whitacre & Nickerson, 2016).

This paper, taking into account the above and specifically that the way of teaching plays a decisive role in the understanding of mathematical concepts and in the development of a positive attitude towards mathematics, presents the results from the application of the RhodeScript theory,

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as it was designed by Avgerinos, Vlachou & Remoundou (2018), which was implemented in the form of didactic interventions by the researchers to students of the 5th grade of the Primary School on the concepts of Geometry, with the aim of improving the students' cognitive potential and mathematical literacy.

2. THEORETICAL FRAMEWORK

2.1 Applications of RhodeScript Theory: A Literature Review In this section, research is presented that applied the RhodeScript Theory into mathematical exercises. Specifically, Beza et al. (2023) applied in their research parts of RhodeScript Theory in pre-interventional concepts, types of angles and inaccuracies that concerned the 3rd, 5th, and 6th grade of Elementary school respectively. The mathematical tools of the RhodeScript Theory that were used are representations, interdisciplinarity, the history of mathematics, realistic mathematics, spatial ability, and geometric transformations, with the aim of improving the performance of students, strengthening their mathematical perception and the development of their positive attitude towards mathematics. The results from the application of these activities showed that students reduced the difficulties they faced in these concepts, based on the international literature, they actively participated in the educational process, their interest in mathematics was strengthened, process that also strengthen the students' mathematical literacy.

In another research of Charinou et al. (2023), the mathematical tools of the RhodeScript Theory were used, such as the Open Problem, Representations and New Technologies. In particular, this work aimed to present the application of the above mathematical tools and their utilization in the teaching practice and specifically on the concept of Area, with the purpose of improving students' concepts concerning Area. The Theory was applied in real classroom situations, with the subject area of Geometry. Geometry was chosen based on the difficulties that students face in this domain, on the individual concepts and the students' prerequisite knowledge. The results of this research showed that the application of RhodeScript Theory helped students to reduce the cognitive barriers in the examine concept. It was also observed that student's interest in mathematics increased, and students' statements showed a positive attitude towards mathematics. Another research (Psaras, 2020a), which conducted on students of the first grade of elementary school in Greece for two school years, investigates whether the use of activities based on the History of Mathematics during teaching can improve students' mathematical ability and attitude.

The results of this research showed that students gained a positive attitude towards mathematics without any elements of fear and aversion, they recognized their usefulness, and they developed their interest in them, thus enhancing the mathematical perception and mathematical ability of students. The same researcher (Psaras, 2020b) investigates whether the use of activities based on Spatial Ability and Geometric Transformations during teaching can improve students' mathematical ability and attitude. The results of this research were similar to the previous ones. Remarkable was the finding where no variable representing the negative attitude of students to mathematics was associated, in the similarity analysis, with variables relating to tasks on spatial ability and geometric transformations.

2.2 Students' misconceptions about types of triangles

Each geometric shape encloses three fundamental entities, first the definition, second the image which is based on the aesthetic experience and finally the figural concept, which is formed and enriched gradually with the help of the images it associates the mind of a person with said shape. Students may present weaknesses in the intake and understanding of any of these three entities. Since one supports the other, problems concerning the image (image) for example, will be transferred respectively to the definition (definition) when the student tries to describe the properties of the shape (Koleza, 2009). Studies of 5–6-year-old children show that children understand the concept more easily if they are able to identify the prototype of the triangle, but have difficulty identifying triangle types, especially if the triangle is inverted or rotated (McIntosh and Draper, 2001: 556).

There were two main misconceptions identified among students regarding the concept of a triangle. The students' first understanding is that a triangle as a mental figure is one that has equal sides and angles. Therefore, more than half of the students (59%) each time they drew a triangle made an equilateral triangle. The second perception concerns a very strong mental image that one finds similarly engraved in the majority of students and is that the base of the triangle must be horizontally drawn (Ayten et al., 2018). For all those theoretical objects (such as the triangle) forms of representation such as drawings, symbols, shapes, verbal, and visual representations need to be used by the students in order to understand its meaning and characteristics. Thus, the students, having initially approached the geometric figures holistically from the small classes of the Primary School, gradually arrive at the management of these shapes which they use as a basis for their reasoning and solving geometry problems in the older classes (Mammana, 1998).

2.3 RhodeScript Theory

This work is oriented towards a new didactic framework in compulsory education, a theory of 10+1 points to strengthen students' mathematical perception and attitude towards mathematics (Avgerinos, Vlachou, Remundou, 2023:3). That is, a toolbox is offered through this theory to the upcoming or active teacher so that he is able to design and implement teachings that promote mathematical knowledge and engage the student. The teaching framework was named “RhodeScript”, a word derived from the initial letters of the names of mathematical tools in English.

1. Representations
2. History of mathematics
3. Open problems
4. breach of Didactical contract
5. Estimation and mental Computation
6. Spatial ability and geometric transformations
7. Counterexamples
8. Realistic Mathematics Education
9. Interdisciplinary
10. Problem posing

+1. Technology

RhodeScript Theory aims to empower mathematical literacy through a variety of practices, methods and tools that prompt students to understand mathematical concepts differently within situations that are meaningful to them, to engage in knowledge discovery processes by externalizing and sharing multiple strategies to solve problems (Avgerinos, Vlachou & Remoundou, 2023).

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In particular, in the teaching interventions of this article, six of the eleven mathematical tools of the RhodeScript Theory were used to design the teaching interventions that were implemented. More specifically, we are going to talk about Representations, Realistic Mathematics, Information and Communication Technologies, Geometric Transformations, Counterexample and Open Problem.

More precisely, every mathematical concept can be visualized, modeled and therefore interpreted with a representation. In this way, students also express themselves using representations to externalize their mathematical reasoning, but at the same time, they manage representations in order to build knowledge and ultimately grasp complex concepts (Coulombe & Berenson, 2001). Two types of representations are distinguished by the researchers of the Teaching Psychology of Mathematics, the external representations, and the internal representations (mental or internal representations). External representations include all those elements that visually represent a mathematical concept or process, such as symbols, shapes, diagrams, images in a physical, digital environment, etc. Internal representations relate to the mental images one mentally constructs to represent a mathematical concept (Golding, 1998; Janvier, 1987a; Janvier, 1987b). Learning results from the two-way interaction of internal and external representations.

Realist Mathematics is the philosophical view of didactic learning introduced by Hans Freudenthal (1973, 1983) that emphasizes the importance and necessity of relating mathematics to reality. Learning must be constructed by the students themselves and agree with reality, even the students' reality that often stems from their fantasy world as well (Avgerinos & Skoufi, 2007). The problems and fields of knowledge and situations in which students are involved should be experientially real to them. To encounter, for example, mathematics in matters of daily life through goods, products, materials, or processes, thus mathematizing their decisions and actions, assessing results and perspectives.

Information and Communication Technologies significantly enhance the educational process, offering new possibilities to teachers by creatively framing each cognitive field. One such scientific area in which information and communication technologies are integrated is mathematics (Sinclair et al., 2006). For example, ICT in geometry can be utilized through the multiple dynamic geometry software offering a competent visualization of the mathematical model, so that the concept, shape characteristics and properties can be understood. Similar use can be made in the collection, processing and analysis of data required for solving mathematical problems, extracting graphs and diagrams of relationships.

Geometric Transformations are understood as the solid movement of shapes that have the property of maintaining the distance between any two of their points in the plane or in space. Euclidean transformations or isometries preserve the property of the geometric object's invariance in any change it undergoes in position or orientation (Hatzikyriakou, 2019).

An important element of a mathematical teaching is the category of Counterexamples. A counterexample is an example that shows that a hypothesis that has been made does not hold. A single counterexample is sufficient to refute a conjecture (Klymchuk, 2001). In fact, with their use in teaching, cognitive destabilization and conflicts are created, therefore the mathematical concept is grasped in a more direct way and knowledge is thus progressively constructed (Klymchuk, 2012).

Finally, the Open Problem is the last teaching tool used by the RhodeScript theory for the planning and implementation of teaching interventions in schools. According to the criterion of

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the possibility of solving a problem, a problem can be considered open, when it includes multiple possible answers and not just one (Pehkonen, 1995).

Solving and constructing open-ended problems by students presupposes advanced cognitive requirements, such as the ability to synthesize and analyze, rational thinking, and competent intake and management of linguistic and mathematical notation.

3.THE RESEARCH

3.1 Research necessity and originality

The need to investigate students' perceptions of geometry as well as the need to ascertain the age at which they should be taught are deemed necessary. Curricula must be updated and adapted to the needs and beliefs of students about the subject matter in order to maximize the likelihood of successful learning. This mathematical field is included in the New Primary Education Mathematics Curriculum and is considered one of the most basic fields.

It has been established through the international literature that for students' understanding of a mathematical concept, the following 4 external factors play an important role, among others: (a) The way mathematics is taught, (b) The use, or not, of mathematical representations concept during its teaching, (c) The textbooks and (d) Teachers' perceptions of the various mathematical concepts (Avgerinos & Vlachou, 2018). In this research, the focus is on the first axis, which concerns the way of teaching. Through the appropriate design and implementation of a teaching intervention in the classrooms, it was observed that the students' performance and attitudes towards mathematics were modified and received a positive sign.

It is therefore necessary to present those practices that succeeded in strengthening the field of knowledge, skills, and attitudes of students as well as their results, in order to inform potential or active teachers around the world of successful teaching tools and methods with positive results, but also to create a fruitful reflection and reflection which revolves around the organization and way of teaching mathematics in primary education.

3.2 Research's aims and objectives

The purpose of this research is the design and implementation of appropriate teaching interventions for mathematics in the 5th grade of elementary school in concepts and projects of the curriculum of the Greek New Curriculum for Mathematics in Primary Education using the tools of RhodeScript Theory, with aimed at reducing the difficulties of mathematics on the examined mathematical concepts and developing a positive attitude towards mathematics.

The ultimate goal of this research is to highlight good practices that strengthen the mathematical knowledge and positive attitude of the students towards this scientific field. At the same time, it is important to overcome the cognitive obstacles presented by students in mathematics, according to the relevant literature, through proposals and tools that have brought positive results.

3.3 Methods

The research followed a qualitative, quantitative approach. Moreover, a content analysis and case study were carried out. Thus, a triangulation was formed, which was methodological, temporal, topical and theoretical in order to achieve stabilization of findings (Cohen et al. 2011).

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3.4 Sample

The sample of this research was 24, 5th grade, students from a primary school in Rhodes, Greece. The hours of the teaching application were 4 teaching hours. The dates of the teaching interventions were between 24/4/2023 to 5/5/2023.

3.5 Research Tools

The research tools used were the teachings, evaluation sheets, semi-structured interviews, and Lesson Study (Lewis, et al., 2009).

3.6 Data analysis

To analyze the survey data, and in addition to the descriptive analysis, the Statistical Implicative Analysis by Gras, using the CHIC (Cohesive Hierarchical Implicative Classification) software (Gras et al., 1997) and Microsoft Excel program were used. The implication analysis of data was performed through similarity diagrams, in which the variables were associated with each other depending on the similarity or non-similarity they present. Variables in whose solution the subjects behave similarly are grouped together.

3.7 Variables of research

The eleven variables were defined as a combination of letters and one number. The letters indicate the initial of concept which is examined. For example, the variable STEx1a is composed of the initial proposal “Select Triangle, Exercise” and number 1a indicates the question of questionnaire. According to the implicative analysis, equivalent to a value of 1 was assigned to every item if the answer is correct and 0 if the answer is wrong or missing.

4. INSTRUCTIVE INTERVENTIONS

4.1 Didactic interventions for Grade 5 of Elementary School: Types of Triangles

This chapter presents the didactic interventions in the 5th grade of elementary school for geometry using the mathematical tools of the RhodeScript Theory, as they have been researched and applied by Avgerinos et al. (2018, 2023), with the aim of improving student performance in the concepts under consideration. The teaching interventions presented concern the phase of control, initiation, discovery, consolidation, and expansion. For each project described below, the phase of the teaching intervention in which it was implemented, the mathematical tools of RhodeScript Theory that were used and its description are mentioned.

4.2 Connect / check project 1: Triangle what do you look like?

Mathematical tools of RhodeScript Theory: Representations, Realistic mathematics, ICT, Geometric transformations

Description of Student Activity - Teaching Management Suggestion: In the first stage of teaching, in what we call Connecting - Checking, the teacher shows in a power point presentation shape that are triangles and others that look like triangles but are not (Figure 1). He then asks students to say whether the shape they see each time is a triangle, giving a brief reason for their answer. In this way, the students' previous perceptions are detected, and the teacher understands their cognitive background in time.

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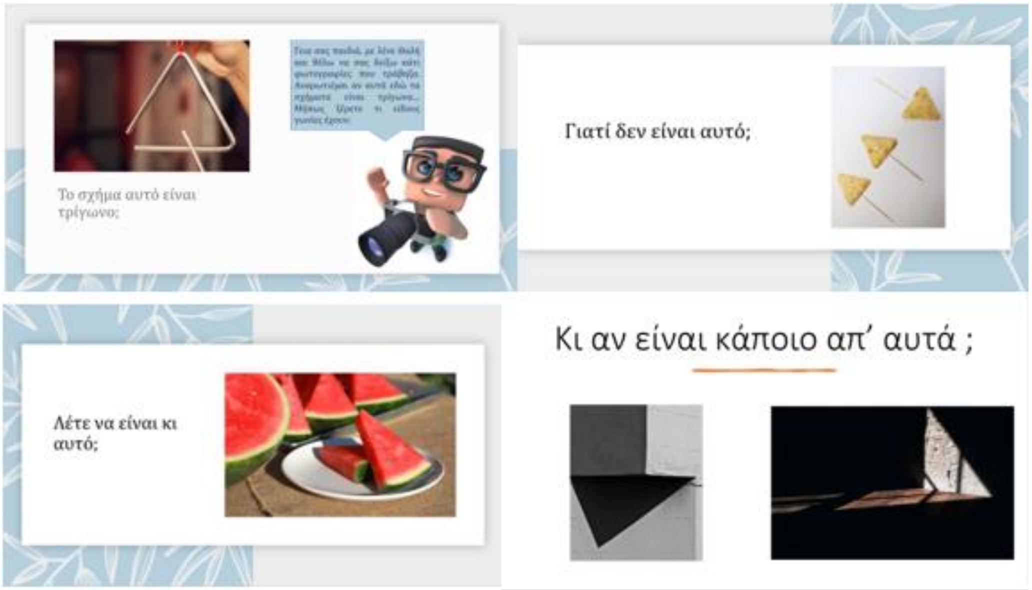

Figure 1. Sample images shown in project 1.

4.3 Discovery Project 2: Fold - unfold and tell me what you see…
Mathematical tools of RhodeScript Theory: Representations, Geometric transformations
Description of student activity - Teaching management proposal: The stage of discovering new knowledge is the most important of all teaching. Students will engage in an investigation in their small groups in which they will need to construct triangles. Specifically, each group will first be given a square, from which an isosceles triangle will be formed by folding the diagonal. Then, a rectangle will be given from which a stair triangle will result if it is folded on the diagonal. Finally, each group will be given a hexagon from which an equilateral triangle will result if the opposite vertices are joined, and a folding is performed on the straight segments that join them (Figure 2). So, the students will successively construct all three types of triangles with their group, and we will record on the board what they observe after constructing each of them.

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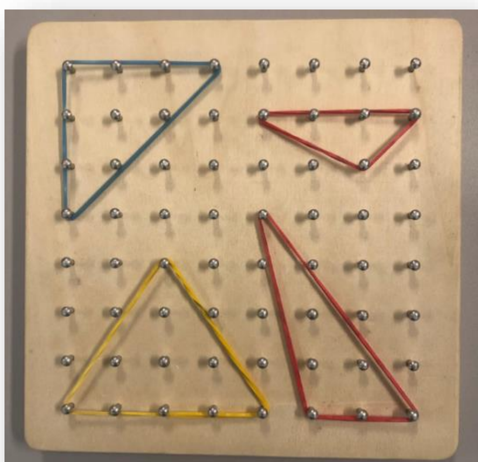
Figure 2. Square, hexagonal, and rectangular cardboard

4.4 Extension Project 3: Rubber band triangles run for the Pin Board
Mathematical Tools of RhodeScript Theory: Representations (Virtual, Symbolic, Verbal), Counterexample, Open Problem. Description of student activity - Teaching management proposal: Students construct triangles with the given rubber bands of different or same colors, using them as sides. These rubber bands are placed on the Pin Board given to them (Figure 3). Then, they record on the table, which is displayed in the classroom, what kind of triangles they made (e.g., right-angled, isosceles). Finally, they are given a statement that they must find if it is true or false, and since the statement is not true, they must prove it with a counterexample, for example, is it true that a triangle can be constructed that is both obtuse and isosceles at the same time?

Figure 3. Pin board, rubber bands and triangles made by students in project 3.



5. RESULTS AND FINDINGS

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5.1 Descriptive statistics

After the teaching interventions, an improvement in students' performance was observed from data analysis. More specifically, from Table 1 we observe that 67% of the students answered correctly the first question concerning the choice of two triangles for the surface of a square with only two triangles (STEx1a). The students seem to have struggled to choose two triangles again but to cover the area of a parallelogram, as 50% of the students answered the related task (STEx1b) correctly.

In the project that asked the students to construct a triangle KLM with side KLM=4cm. and each of the angle's K and L to be 60° (DATEx2) 83% of the students answered correctly. We had the same success rate in the task of classifying triangles in terms of angles and sides. More specifically, in the classification of the equilateral triangle in terms of angles, 83% of the students answered correctly (TCaAEx3), while all students answered correctly in the classification of the same triangle in terms of sides (TCaSEx3). Also, all students correctly answered the classification of given isosceles triangle by angles (TCbAEx3) and given isosceles, right, and obtuse triangles by sides (TCbSEx3, TCcSEx3, TCdSEx3).

We notice that the percentages decrease in the classification of the above triangles based on their angles. More comparatively, 67% of students correctly answered the task that asked them to find the type of triangle in terms of the angles of a given right triangle (TCgAEx3) and 83% of students correctly answered the task that asked them to find the type of triangle in terms of angles of a given obtuse/isosceles triangle (TCgAEx3).

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	Occurrence	Average	Standard deviations:
STEx1a	: 16.00	0.67	0.47
STEx1b	: 12.00	0.50	0.50
DATEx2	: 20.00	0.83	0.37
TCaAEx3	: 20.00	0.83	0.37
TCaSEx3	: 24.00	1.00	0.00
TCbAEx3	: 24.00	1.00	0.00
TCbSEx3	: 24.00	1.00	0.00
TCgAEx3	: 16.00	0.67	0.47
TCcSEx3	: 24.00	1.00	0.00
TCdAEx3	: 20.00	0.83	0.37
TCdSEx3	: 24.00	1.00	0.00

Table 1. Frequency of student responses to the assessment phase questionnaire projects

5.2 Similarity Tree

The implication analysis of data was performed through similarity tree. In similarity tree (Fig. 4) the variables were associated with each other depending on the similarity or non-similarity they present. Variables in whose solution the subjects behave similarly are grouped together. According

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to similarity tree (Fig. 4), which presents the task after the instructive interventions, it is possible to distinguish observe three groups.

The first group (group A) ((STEx1a, STEx1b), (TCbSEx3, TCgAEx3)) includes the variables related to the projects concerning the covering of square and parallelogram surfaces after selecting two suitable triangles (STEx1a, STEx1b) and the variables that related to the isosceles and right triangle classification projects (TCbSEx3, TCgAEx3). These variables apart from TCbSEx3 belong to the projects that made it more difficult for the students, as they gathered the lowest success rates.

The second group (group B) ((DATEx2, TCaAEx3), TCdAEx3)) includes variables concerning projects in which students construct triangles (DATEx2) and classify triangles according to their angles (TCaAEx3, TCdAEx3). From this connection of similarity, it can be seen that the students who managed to draw a triangle KLM with side KLM=4cm. and each of the angle's K and L to be 60° (DATEx2), they were able to correctly name the equilateral (TCaAEx3) and isosceles/obtuse triangle (TCdAEx3) based on the type of their angles.

The third group (group C) ((TCaSEx3, TCbAEx3) (TCcSEx3, TCdSEx3)) mainly includes variables related to projects in which students classify triangles based on sides and are the projects in which students scored the best. It seems, that is, that the classification of triangles based on the sides makes it less difficult for students than the classification of triangles based on the criteria of the angles.

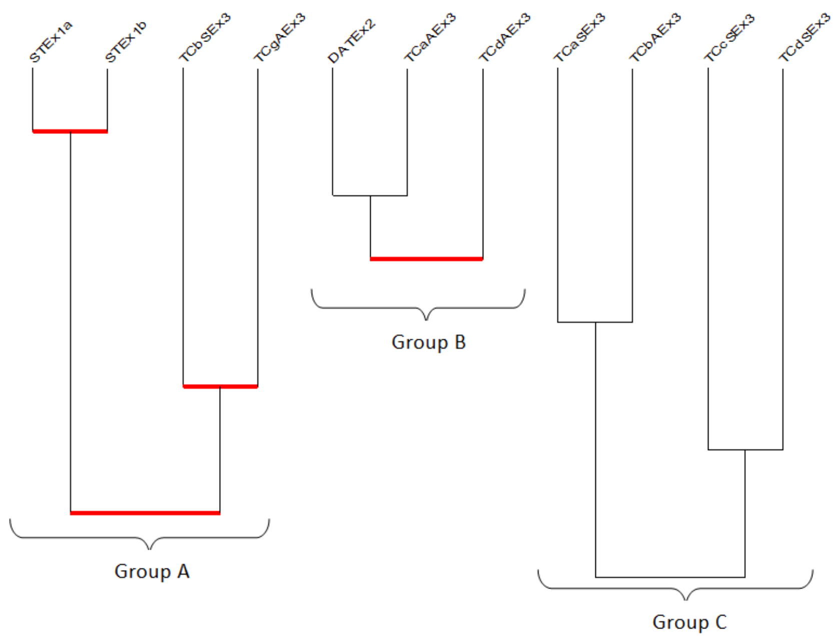

Figure 4. Similarity diagram of student responses to the assessment phase questionnaire projects

6. CONCLUSION AND DISCUSSION

This research presents the results from the application of the RhodeScript Theory in the teaching of mathematics in Primary School, with the aim of strengthening and improving mathematical literacy. In particular, this paper examines the difficulties faced by 5th grade students in Geometry subjects and specifically in the classification of triangles based on their angles and sides and seeks to reduce the cognitive barriers that appear in these concepts, according to the relevant bibliography, through didactic interventions using the mathematical tools of RhodeScript Theory.

Regarding the types of triangles, the 5th grade students scored high in the assessment and practice projects and developed a positive attitude towards geometry. The students abandoned their misconception that a triangle is the shape that has equal sides and angles precisely because they were asked to draw many different triangles of varying sides and angles. At the same time, the strong mental image they initially had of the horizontal base of the triangle was deconstructed, as students gradually began to draw triangles with different orientations, making transformations such that the base of the triangle was not drawn horizontally.

An interesting proposal arising from the present research would be its extension to different thematic centers of the field of mathematics. For example, there would be value in designing and implementing more instructional interventions for students in these elementary grades that address subject areas such as algebra, statistics, measurement, and probability. In this way, the tools of RhodeScript theory will be used in a pleasant and beneficial way and the New Math Curriculum of 2022 will be assisted, since applications suitable for its substantial promotion and all-round diffusion will be designed.

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