International Journal of Education Humanities and Social Science

ISSN: 2582-0745
Vol. 6, No. 04; 2023

USING THE APPLICABLE THEORETIC FRAMEWORK RHODESCRIPT ON MATHEMATICS OF THE PRIMARY EDUCATION: THE CASE OF MEASUREMENT AND GEOMETRY

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https://doi.org/10.54922/IJEHSS.2023.0540

ABSTRACT
This paper addresses the difficulties faced by students in the topics of Measurement and Geometry and presents activities designed using the mathematical tools of the applicable theoretic framework RhodeScript, which were successfully implemented during practical exercises in primary schools in Rhodes, Greece. Specifically, the mathematical concepts upon which the lessons were designed include pre-area concepts, types of angles, and symmetry, targeting students in the 3rd, 5th, and 6th grades of primary school, respectively. The mathematical tools utilized from the RhodeScript Theory include representations, interdisciplinarity, history of mathematics, realistic mathematics, the breach of the didactical contract, and geometric transformations, with the aim of improving students' performance, enhancing their mathematical perception, and fostering a positive attitude towards mathematics. The results from the implementation of the activities showed that the students reduced difficulties they encountered in these concepts based on the international literature, actively participated in the educational process, and their interest in mathematics was enhanced, processes that enhance mathematical literacy.

Keywords: Applicable theoretic framework RhodeScript, measurements, geometry, primary education.

1. INTRODUCTION
Students in many countries worldwide, including Greece, exhibit very low performance in international assessment programs (PISA, 2003; TIMSS). The difference in performance between students from Asian countries, primarily Singapore, achieving the highest results, and students from African countries such as Egypt and South Africa, achieving the lowest results, as well as students from European countries like Italy and France, achieving moderate performances, is concerning. The performance of Greek students in mathematics is equally worrisome (PISA, 2015).

Taking into account the Greek data on mathematics education, this study presents applied practices using the tools of the RhodeScript theory within the framework of practical training for graduating students and future educators in primary schools in Greece. Specifically, the purpose of the teaching interventions was to strengthen students' mathematical understanding and develop positive attitudes towards mathematics, as well as to alleviate difficulties faced by students in the thematic areas of measurements and geometry, as presented in the international literature. In this study, selected interventions are chosen and presented, which have achieved the highest success based on their implementation and evaluations.

ISSN: 2582-0745
Vol. 6, No. 04; 2023

2. THEORETICAL FRAMEWORK

2.1 Difficulties of students in taught concepts

According to the international literature, primary education students face difficulties in the process of estimating the area of a surface (Myrovali, 2016) and generally in spatial estimation, resulting in limited spatial ability and perception. They also consider Geometry to be one of the most challenging subjects (Kalaitzidis & Pappa, 2011).

Regarding the concept of angles, students struggle when asked to evaluate the size of an angle. They tend to focus on various other elements of the angle, such as the length of the half-lines (Papadopoulos, 2019).

In terms of symmetry, students find it challenging to comprehend the term "axial symmetry," which involves the ability to recognize symmetry within a shape and the ability to construct a symmetrical figure. The ability to recognize precedes, naturally, for students to understand the purpose of the construction process. Axial symmetry is relatively easy to recognize, but its construction is a complex and problematic procedure (Kassioti et al., 2009)

2.2 The Applicable Theoretic Framework RhodeScript

The activities conducted during teaching in primary schools in Rhodes, presented in this study, are based on the use of mathematical tools from the Theory of RhodeScript, as they have been researched and applied by Avgerinos et al. (2018).

The Mathematics Education and Multimedia Laboratory at the University of the Aegean organizes and supervises (since its establishment in 2000) the theory and practice of Applied (and essentially Applicable) Mathematics Education for graduating students of the Pedagogical Department, as well as prospective teachers, through Practical Exercises in two phases and the presence of students in real classroom conditions in the primary schools of Rhodes. Taking into account the experience of so many years, the research team of the Laboratory proposes, based on observations of students and educators, an approach framework for teaching and handling the content of mathematics in real classroom conditions, considering all previous reflection, the Applicable Theoretic Framework RhodeScript (Avgerinos et al., 2023).

The Applicable Theoretic Framework RhodeScript is a theory based on eleven fundamental mathematical practices, or rather, on 10+1 tools. The instructional framework was named "RhodeScript," a word derived from the initials of the names of the mathematical tools in English.

1. **R**epresentations
2. **H**istory of mathematics
3. **O**pen problems
4. Breach of **D**idactical contract
5. **E**stimation and mental computation
6. **S**patial ability and geometric transformations
7. **C**ounterexamples
8. **R**ealistic Mathematics Education
9. **I**nterdisciplinarity
10. **P**roblem posing

ISSN: 2582-0745
Vol. 6, No. 04; 2023

+1. **T**echnology

The RhodeScript Theory aims to empower mathematical literacy through a variety of practices, methods, and tools that encourage students to understand mathematical concepts in different ways within meaningful situations for them. This allows them to engage in knowledge discovery processes by externalizing and exchanging multiple strategies for problem-solving (Avgerinos et al., 2023).

Specifically, in the pedagogical proposals of this article, representations were utilized as a mathematical tool. There are two types of representations: external and internal representations (Lesh, Post & Behr, 1987). In this work, we will only focus on external representations (tangible materials), which involve the live representation and visualization of mathematical concepts in real-time to make them understandable and more accessible to students.

The breach of the didactical contract is an important mathematical tool since school mathematical problems share many similarities in terms of presentation and solution methods (Brousseau, 1981). Students need to be equipped with all the necessary knowledge and skills to solve a mathematical problem that is completely different from the usual textbook problems. Therefore, disruptive and critical thinking about mathematical problems and the multidimensional perspective of students on them are the key to breaking the teaching contract.

Additionally, realistic mathematics is another mathematical tool used in this work. Realistic mathematics is a pedagogical and learning philosophy conceived by Freudenthal (1973, 1983) who believed that mathematics is a human activity that must be connected to reality, influenced by and influencing society, and made as understandable as possible for students.

The history of mathematics is also used as a mathematical tool. The history of mathematics can be used in education either as a tool or as a goal (Jankvist, 2009a, Jankvist, 2009b). As a tool, history helps in the learning of mathematics by providing motivation, while in the second case, history becomes the goal of learning.

Another mathematical tool utilized is interdisciplinarity, as the integration of one or more disciplines is particularly beneficial during the educational process (Deneme & Ada, 2012). Finally, geometric transformations are used as another tool in this work. They refer to solid motions, i.e., geometric transformations in the plane or space that do not change the lengths of lines or the measures of angles under which they meet, while preserving isometries and distances between any two points (Yaglom, 1962, 2009).

3. RESEARCH

3.1 Purpose of the research

This study aims to address the difficulties that students face in understanding the concepts of area, angles, and symmetry. Using the mathematical tools of the RhodeScript Theory, the goal is to reduce these difficulties for students. Specifically, the objectives of the study are to investigate:

1. To what extent the application of the RhodeScript Theory in teaching area can enhance students' geometric thinking through practice in composition and analysis of shapes.

ISSN: 2582-0745
Vol. 6, No. 04; 2023

2. Whether the application of the RhodeScript Theory in teaching different types of angles can help overcome students' difficulty in evaluating the size of an angle, by not focusing on various elements of the angle, such as the length of the half-lines.
3. How the application of the RhodeScript Theory in teaching symmetry can leverage students' pre-existing knowledge of symmetry, which is familiar to them from their environment, and approach activities based on the geometric understanding of symmetry by students, including the properties of shapes and formations.

4. METHODS MATERIALS

4.1 Sample

The population of the research consisted of 65 students from the 3rd, 5th, and 6th grades of two primary schools in Rhodes, Greece.

4.2 Research Tools

To achieve the research objectives, instructional interventions were conducted, totaling 6 teaching hours, in April and May 2023. Qualitative data collection was carried out through observation and the use of a journal, which involved written depiction of events during the teaching process.

5. TEACHING INTERVENTIONS

In this chapter, the selected works of the teaching interventions are presented, specifically four activities on geometric formations using tangrams, types of angles, and symmetry for the 3rd, 5th, and 6th grades of primary school, respectively. These activities were implemented using the mathematical tools of the applicable theoretic framework RhodeScript, as researched and applied by Avgerinos et al. (2018), with the aim of improving students' performance and fostering a positive attitude towards Mathematics.

The proposed activities serve different phases of teaching. The first activity is part of the discovery phase. The second activity is part of the extension phase. The third activity also falls within the extension phase, and the fourth activity is in the initiation phase. These activities belong to different phases because they originate from different teaching interventions, and only the best activities from those interventions are presented.

5.1 Activity 1

3rd Grade of Primary School - Geometry: Getting to Know Tangrams

Objective: Students recognize and name two-dimensional geometric shapes, practice analyzing a composite shape into its constituent shapes, and compose and decompose two-dimensional shapes into other two-dimensional shapes.

Mathematical tools of RhodeScript Theory: Representations, History of Mathematics, Geometric Transformations, Interdisciplinarity

Description of student activity - Proposed teaching approach: Through discussion, it is mentioned that Tangrams are a Chinese game, specifically a puzzle consisting of 7 pieces. After providing information about this Chinese game, questions are asked to the students regarding the shapes they see, in order for them to recognize and name them and identify which part of the whole each piece of the tangram represents. At this point, transformations and movements of the pieces begin, allowing students to understand the shapes and the relationships between them. For example, students are asked to isolate the square and the small triangles and then create a square

ISSN: 2582-0745
Vol. 6, No. 04; 2023

using the two small triangles (Figure 1). The process continues with the other shapes, such as a large triangle and the corresponding two small triangles and square (Figure 2). It is desirable for the triangles of the tangram that have different sizes to also have different colors to facilitate the activity.

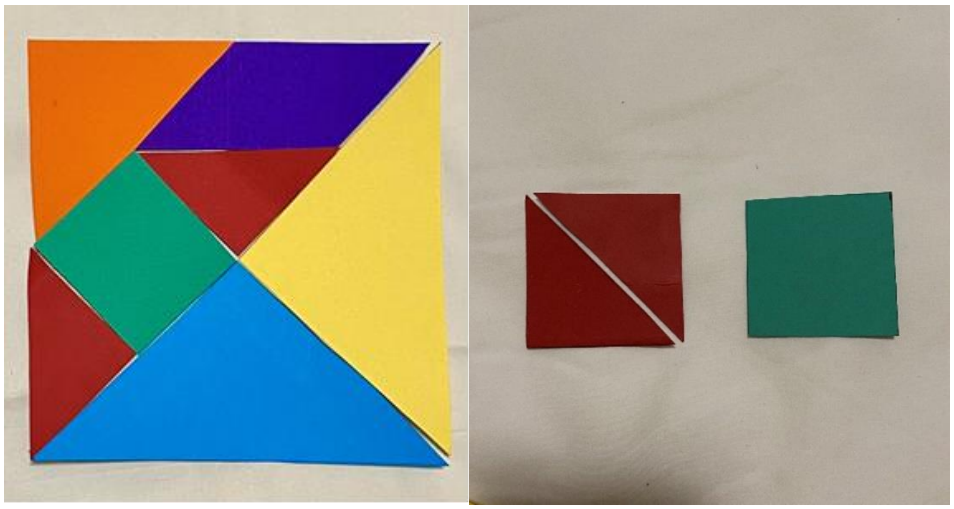

Figure 1: Indicative image from the implementation of Activity 1.

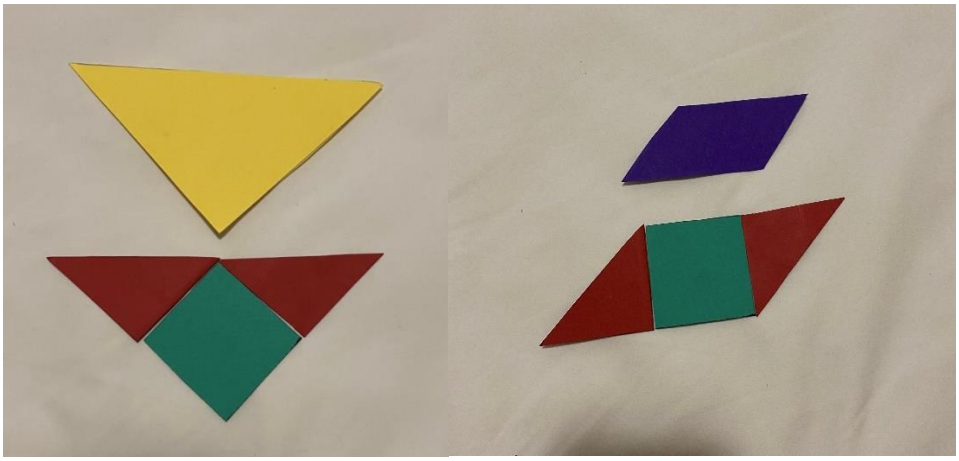

Figure 2: Indicative image from the implementation of Activity 1.

5.2 Activity 2

5th Grade of Primary School - Measurements: Create angles with your body

Objective: Students to distinguish between types of angles (right, acute, obtuse, straight) and compare and construct angles.

Mathematical tools of RhodeScript Theory: Realistic Mathematics, Geometric Transformations.

Description of student activity - Proposed teaching management: In this stage of the mathematical work in the classroom, we divide the students into groups and ask them to create angles with their bodies, for example, using their hands or feet (Figure 3). In each round, one student from each group stands up, and each student must create the type of angle assigned to them

ISSN: 2582-0745
Vol. 6, No. 04; 2023

without repeating an angle created by another student before (Figure 4). With each correct answer, the group earns a point, and the group with the most points wins.

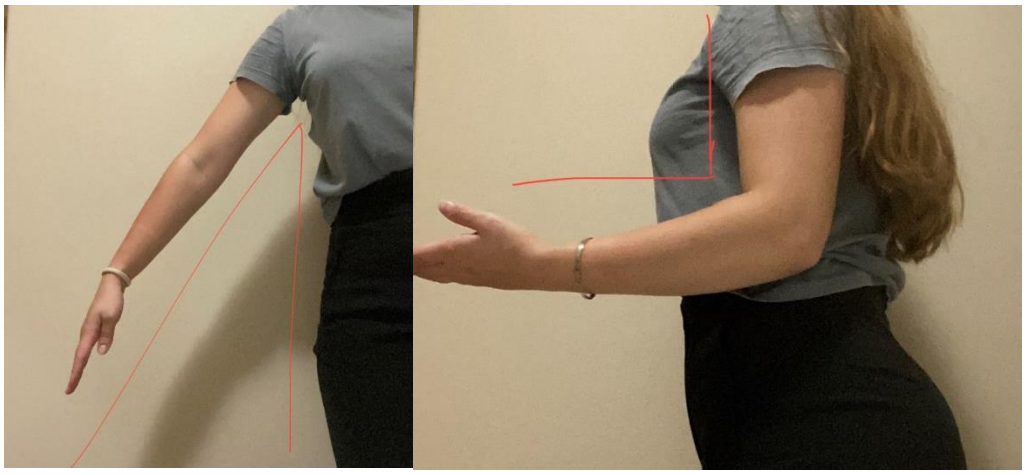

Figure 3: Indicative image from the implementation of Activity 2.

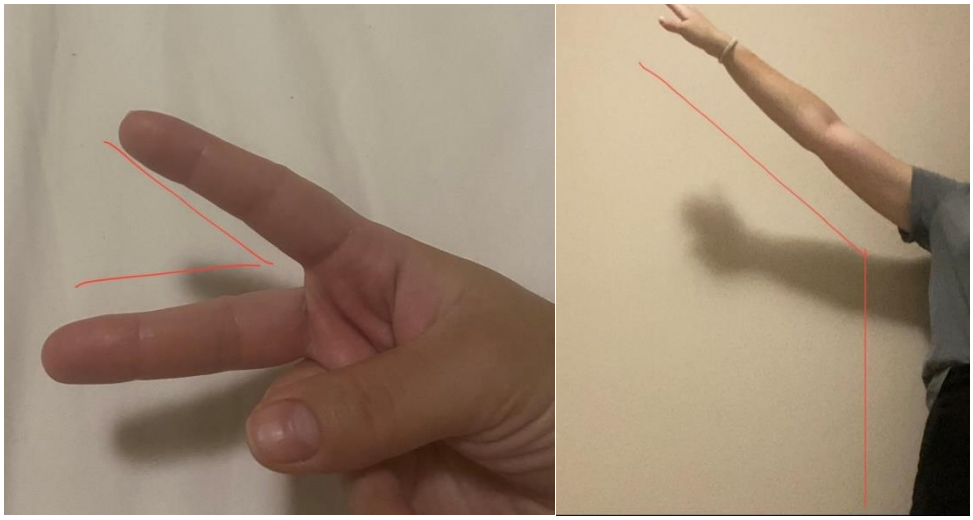

Figure 4: Indicative image from the implementation of Activity 2.

5.3 Activity 3

6th Grade of Primary School: Geometry - Discovering Symmetry

Objective: Students to develop mathematical creative criticism and discover the desired knowledge of symmetry.

Mathematical tools of RhodeScript Theory: Representations, Geometric transformations, Breach of Didactical contract

Description of student activity: For this activity, various geometric shapes have been prepared at home using paper (Figure 5), which were distributed to the entire class. Each student receives an A4 sheet with the same shape, and they first have to identify the shape and then cut it out following the outline. After all the students have cut out their shapes, they need to fold them in such a way that when they unfold them, the resulting line divides the shape symmetrically. This

ISSN: 2582-0745
Vol. 6, No. 04; 2023

will be done individually, and at the end, each student will present their result. This process will be repeated with three shapes: a circle, a square, and a triangle. This makes the learning process more engaging, and understanding axial symmetry becomes easier as students witness it happening in reality.

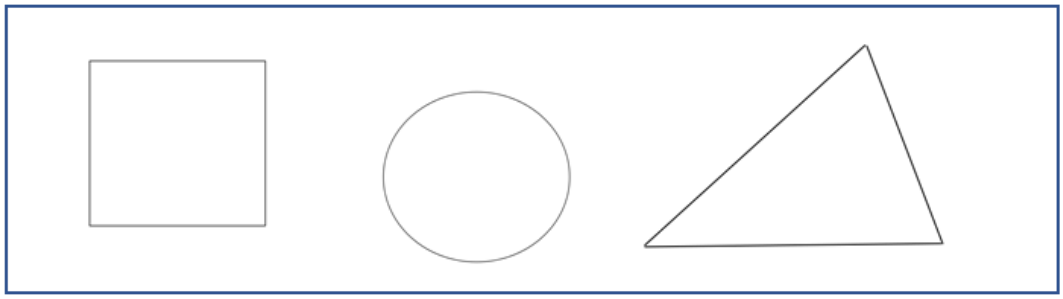

Figure 5: Indicative image from the implementation of Activity 3

5.4 Activity 4

6th Grade of Primary School - Geometry: Symmetry around us
Objective: Students to observe that nature itself is composed of symmetry.
Mathematical tools of RhodeScript Theory: Realistic Mathematics, Geometric Transformations
Description of student activity: In this activity, we will ask students to observe their surroundings and find objects that exhibit symmetry. In this way, they will understand that symmetry applies to various things, whether they are geometric shapes, everyday objects, animals, insects, or even parts of our bodies. We will ask the students to divide into groups of two and they will be asked to observe their classmate's face. Then they should find the face that consists of symmetrical parts and can be divided by a line to form an axial symmetry.

6. RESULTS

The results obtained from the above projects are that during the implementation of Activity 1, which focused on the concept of geometric transformations using the puzzle Tagram, it was observed that the students had a better understanding of the transformations of geometric shapes. The students had to operate using their geometric thinking to better comprehend the geometric shapes and their areas. They would take the shapes, move them, rotate them, and position them in various ways to achieve the desired result. For the creation of the large triangle, for example, many children would take the medium triangle, rotate it, combine it with other pieces, and realize that it was larger than necessary.

During the implementation of Activity 2, which focused on the concept of types of angles using realistic mathematics and geometric transformations, it was observed that the students understood the types of angles and that the type of each angle depends solely on the opening of its sides. The students used their creativity to constantly think about how they could construct other angles using their bodies. They created angles using the openings of their legs, arms, fingers, assuming positions with their bodies such as sitting, and more.

During the implementation of Activity 3, which focused on the concept of symmetry using representations, realistic mathematics, and the breach of didactical contract, it was observed that all students actively participated in the activity. The circle was the easiest geometric shape given

ISSN: 2582-0745
Vol. 6, No. 04; 2023

to them since no matter how they folded it, the axis of symmetry would be correct. The square required more imagination from the students as there was not only one way to create symmetry. Most students folded the square in the same way initially, dividing it in half into two congruent rectangles. Then they were asked to find another way, namely by dividing it into two congruent triangles. Finally, the triangle was scalene and did not have an axis of symmetry, which the students had to understand, and most of them did. The selection of this shape was made so that stereotypical perceptions would not be created for the students when solving exercises, but rather to process the exercise data each time and then continue the search for a solution if there is one.

During the implementation of Activity 4, which also focused on the concept of symmetry using realistic mathematics and geometric transformations, it was also observed that the students actively participated in this activity. The correlation of the mathematical concept with the real world made the students more interested in participating in the lesson and better understand the mathematical concept being taught. The students did not stop giving answers related to symmetrical things and objects inside the classroom, such as the board, desks, the door, the clock, chairs, hangers on the wall, the trash can, and more.

7. DISCUSSION-CONCLUSIONS

In this study, four activities were successfully implemented in three elementary school classes 3rd, 5th, and 6th grades, using mathematical tools from the applicable theoretic framework RhodeScript (Avgerinos et al, 2018, 2023). Specifically, the activities involved representations, realistic mathematics, the history of mathematics, geometric transformations, the breach of didactical contract, and interdisciplinarity, aiming to enhance students' understanding of the taught mathematical concepts. The results from the implementation of the activities showed that the students reduced difficulties they encountered in these concepts based on the international literature, actively participated in the educational process, and their interest in mathematics was enhanced, processes that enhance mathematical literacy.

To improve students' mathematical literacy and enhance their performance in mathematics, it is necessary to adopt beneficial educational practices. Of course, students' performance is influenced by various factors such as the socio-economic conditions prevailing in their environment, their family background, the educational framework of teaching and learning, the training of educators, the equipment of educational units, and even the effects of gender (Tokatlidou & Theodosiou, 2020). However, what can certainly be improved through education is the adoption of methods, practices, and tools that help students understand mathematical concepts through meaningful situations for them, thereby strengthening their mathematical thinking and attitude towards mathematics. In addition, the research was done on a small number of sample and cannot make generalizations, although the findings of present research may help researchers to extend the use of applicable theoretic framework RhodeScript to other mathematical concepts.

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ISSN: 2582-0745
Vol. 6, No. 04; 2023

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