

**RESEARCH ON THE COGNITION OF PRE-SERVICE HIGH SCHOOL
MATHEMATICS TEACHERS FOR IMPLEMENTING INTUITIVE IMAGINATION
LITERACY**

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ABSTRACT

Currently, intuitive imagination literacy has attracted extensive attention from all walks of life. Many relevant problems about it have been studied except the cognition of implementing intuitive imagination literacy of pre-service high school mathematics teachers. To address this gap, this study investigates 51 pre-service high school mathematics teachers through open-ended interviews to learn about the cognitive situation of pre-service high school mathematics teachers for implementing intuitive imagination literacy. After analyzing, it can be found that about implementing intuitive imagination literacy: 1. Pre-service high school mathematics teachers do not have a comprehensive cognition. Their cognition, both as a whole and in different dimensions, does not reach half of the number of implementation recommendations made by the previous authors, and the subjects they realize for implementation are limited to both teachers and students. 2. Pre-service high school mathematics teachers do not have a very reasonable cognition. Some of their ideas lack feasibility or effectiveness and exist a certain one-sidedness, which needs to be further explored and considered. Therefore, it is suggested that: 1. Experts and teachers who are responsible for educating teachers should increase emphasis on intuitive imagination literacy and provide pre-service teachers with more opportunities to practice teaching. 2. Pre-service teachers should take the initiative to study and research, focus on combining theory and practice, and continuously reflect, adjust and improve.

Key Words: Pre-service teachers, High school, Mathematics, Intuitive imagination, Literacy, Implementing measures.

1. INTRODUCTION

Intuitive imagination is the literacy of using geometric intuition and spatial imagination to perceive the forms and changes of things and using spatial forms, especially graphs, to understand and solve mathematical problems. Intuitive imagination, as one of the six core mathematical literacies, focuses on guiding students to perceive the form and change of things from the perspective of geometric intuition, which is an important part of core literacy. It is also an important means of identifying and posing problems, analyzing and solving them, and a basis for exploring and forming ideas for argumentation, mathematical reasoning, and building abstract structures of thinking (Ministry of Education of the People's Republic of China, 2018). Therefore, it is

necessary to develop students' intuitive imagination awareness and intuitive imagination ability in high school mathematics education, and its mathematical education value cannot be underestimated (Zhang and Pei, 2020). However, many scholars and teachers have extensively discussed and studied the present situation of high school students' cognition of intuitive imagination literacy, then they found that the current level of intuitive imagination literacy among high school students is not high (Du, 2019), which showed that intuitive imagination literacy has not been well implemented in actual teaching. What are the reasons for this? How to develop students' intuitive imagination literacy? This is a question worthy of our study.

2. LITERATURE REVIEW

Currently, there have been a lot of studies on this issue of implementing intuitive imagination literacy in high school mathematics teaching.

2.1 The Situation of High School Students' Intuitive Imagination Literacy Level

In 2018, the General High School Mathematics Curriculum Standards (2017 Edition) (hereinafter referred to as Curriculum Standards (2017 Edition)) specify three levels of intuitive imagination literacy and establish a framework for the corresponding literacy levels. Based on the "pyramidal" assessment framework constructed from the aspects of content, structure, and process, Yin conducted a study on the assessment of high school students' intuitive imagination literacy and found that the level of geometric intuitive imagination was higher than the level of algebraic intuitive imagination (Yin, 2017). Weng found that the overall level of intuitive imagination literacy of high school students is not high, and the overall level is at level 1. Furthermore, the number of students who can reach this level decreases as the level of intuitive imagination literacy increases from level to level (Weng, 2017). Chen found that the overall level of high school students' intuitive imagination literacy was low and students' scores on the three dimensions of spatial imagination, number-shape combination, and geometric intuition were decreasing in order (Chen, 2018). Li further obtained from her study that there are three main deficiencies in the current mathematical intuitive imagination literacy development of high school students: they are unable to make precise identification of mathematical concepts related to intuitive imagination literacy, they are unable to demonstrate solid graphing fundamentals in describing practical problems graphically, and they are unable to solve problems comprehensively with refined mathematical language based on problem characteristics (Li, 2021).

2.2 The Factors Affecting Students' Intuitive Imagination Literacy

Yang found that teachers' professionalism had a significant impact on students' visualization ability. In solving conventional visualizable problems, the difficulty of the question, the presentation, and whether students use schematics are factors that affect students' visualization ability (Yang, 2012). Using a self-administered intuitive imagination literacy test paper to measure students, Zheng et al. found that the complexity of the situation and the presentation of the graphs differed in the level of intuitive imagination literacy required of students; there were significant differences in the level of intuitive imagination literacy between boys and girls, and the level of students' intuitive imagination literacy was uneven across geographic regions (Zheng et al., 2020). After analyzing the current situation and differences in intuitive imagination literacy of high school students, Zhang and Pei further pointed out that, on the one hand, the selection of the research subjects and the choice of research methods had an impact on the results, and on the other hand,

students' intuitive imagination literacy gradually showed gender differences with age, but the gender differences shown by high school students were not yet stable (Zhang and Pei, 2020). Tao found that all dimensions of logical reasoning ability and all dimensions of intuitive imagination ability showed a positive correlation, while logical reasoning ability and intuitive imagination ability were significantly and positively correlated with academic performance in mathematics (Tao, 2021).

2.3 Strategies for Cultivating High School Students' Intuitive Imagination Literacy

Most scholars have focused their attention on this aspect of strategies for developing the intuitive imagination ability of high school students, and their research can be broadly divided into two dimensions: "general teaching" and "classroom teaching".

In the dimension of "general teaching", Yang suggested that the mathematical intuitive teaching should advocate the combination of intuition and speech, including explaining the geometric meaning of abstract mathematical knowledge; constructing the network of verbal and representational representations of mathematical knowledge; and delaying the use of symbolic system (Yang, 2012). Jin suggested that teachers must carefully study the tangents of background materials and the relationship between them and mathematical models in teaching, use them rationally, and guide students to discover mathematical laws and models from them to solve problems (Jin, 2016). From the perspective of PME, Shen and Wang proposed that in the teaching process, teachers should not only pay attention to the mathematical teaching of critical periods, but also provide appropriate prior organizers, stimulate students' metacognitive monitoring and regulation, and design appropriate inquiry-based questions (Shen and Wang, 2017). Li put forward the following requirements for teachers: to carefully study the curriculum standards and clarify the teaching directions and requirements; to broaden the ways, time, and space for teaching intuitive imagination; to strengthen the guidance of intuitive imagination strategies and methods; and to strengthen the practice and perception of students' intuitive imagination (Li, 2019). Chang believes that the penetration of intuitive imagination literacy in mathematics teaching has a triple realm: (1) shape - the ability of straightforward sensory imagination; (2) thought - the formation of the idea of combining numbers and shapes; (3) new - the constructing a system of intuitive models (Chang, 2020).

In the dimension of "classroom teaching", Zhang and Han suggested that students should be guided to observe spatial geometry, make models of spatial figures, accumulate representations of spatial figures, and develop their spatial intuitive imagination literacy through hands-on practice and confirmation (Zhang and Han, 2017). Gu and Wang emphasized cultivating the habit of looking at problems and drawing diagrams, learning to graph complex problems, and also learning to use information software to recognize graphical relationships and form spatial concepts (Gu and Wang, 2020). Liu et al. emphasized the importance of understanding and analyzing problems from a transformational perspective with the help of geometric intuition. They believe that teachers should pay attention to the needs and abilities of students at different levels to develop agile insight into intuition in a gradual manner (Liu et al., 2020). Lin's study on the connection between GeoGebra software and the development of intuitive imagination literacy concluded that it is beneficial for teachers to develop students' intuitive imagination literacy by making reasonable use of software that can demonstrate the generation and dynamic changes of mathematical

elements in mathematics teaching (Lin, 2021). Based on his research, Tao suggested strengthening context creation, emphasizing inductive reasoning, and integrating the teaching of logical reasoning and intuitive imagination (Tao, 2021).

2.4 The Assessment Methods of Intuitive Imagination Literacy of High School Students

The Curriculum Standards (2017 Edition) emphasize that the evaluation of intuitive imaginative literacy should be based on the curriculum objectives and academic quality standards, pay attention to stage, continuity, and wholeness, and focus on the main performance of the four aspects of intuitive imagination literacy (Ministry of Education of the People's Republic of China, 2018). With reference to Bloom's learning assessment model, PISA learning assessment model, and SOLO learning assessment model, Yu proposed the theoretical concept of classifying mathematical literacy assessment into three forms: knowledge understanding, knowledge transfer, and knowledge innovation (Yu, 2017). Zhu constructed a three-dimensional assessment framework of "subject content \times literacy components \times observation indicators" and assigned weight values to the dimensions of mathematical core literacy and observation indicators for high school students to obtain the expressions between mathematical core literacy and the three assessment dimensions of mathematical knowledge, problem-solving, and mathematical thinking (Zhu, 2020). Based on the identification of concepts and theories related to intuitive imagination literacy, Zheng et al. used the Delphi method to construct an intuitive imagination literacy assessment index system consisting of three primary indicators and nine secondary indicators, and then they used a combination of quantitative and qualitative research methods to construct an intuitive imagination literacy assessment model (Zheng et al., 2021).

From the above studies, we can see that many scholars have studied intuitive imagination literacy in terms of its situation, influencing factors, and other aspects, and have especially made a lot of suggestions on the cultivation strategies of intuitive imagination literacy. However, it can be also seen that few people have conducted research on teachers' intuitive imagination literacy, and the research on teachers' cognition of implementing intuitive imagination literacy is in a gap. Through previous studies, it can be found that teachers' cognition of implementing intuitive imagination literacy will affect students' mathematics learning and development to a large extent, so studying this issue is of great practical importance to the implementation of intuitive imagination literacy. Therefore, the purpose of this study is to find out the current pre-service high school mathematics teachers' cognition of implementing intuitive imagination literacy through investigation.

Thus, the main issues of this study are:

1. What areas do the current pre-service high school mathematics teachers focus on regarding the implementation of intuitive imagination literacy?
2. Is the cognition of implementing intuitive imagination literacy comprehensive among current pre-service high school mathematics teachers?
3. Is the cognition of implementing intuitive imagination literacy reasonable among current pre-service high school mathematics teachers?

3. RESEARCH METHODS

3.1 Participants

To faithfully reflect the pre-service high school mathematics teachers' cognition for implementing intuitive imagination literacy, this study used the whole-group sampling method and selected 51 masters of education majoring in mathematics of grade 2021 from the School of Mathematics and Statistics of Shandong Normal University as the survey sample. They include 48 girls and 3 boys, whose ages are distributed between 21 and 26 years old. All of them hold high school mathematics teacher qualification certificates and have the intention to go to high school for employment in the future.

3.2 Instrument

In this study, we conducted open-ended interviews with an interview outline containing a total of 10 questions, of which the two most prominent questions were, "How do you think intuitive imagination literacy can be implemented in high school?" and "How do you think intuitive imagination literacy can be implemented in mathematics classrooms?" The open-ended interview method is adopted because it is fast, convenient, flexible, not restricted by written language, and facilitates in-depth investigations to obtain the most direct information. The reason for choosing the above questions is to find out the pre-service high school mathematics teachers' real cognitive situation for implementing intuitive imagination literacy.

3.3 Data Collection

To ensure the reliability of the research, the open-ended interview method was used to interview 51 masters of education one by one individually, and the interview content was recorded during the whole process after the consent of the other party was sought.

3.4 Data Processing

Firstly, we converted the interview recording content into text form, removed um, ah, and other discourse markers, strictly followed the original words of the interview to sort out. The core ideas they expressed were further extracted and categorized into two primary indicators: "general teaching" and "classroom teaching", and according to the subject of implementation, each primary indicator was divided into two secondary indicators: "teachers" and "students", which were represented by A, B, C, and D respectively. Then we used SPSS to count the number of people mentioned in each of the above items and calculate the corresponding percentages. Finally, the corresponding tables were made based on the data obtained.

4. RESULTS

4.1 Cognitive Focus

The core ideas expressed by pre-service high school mathematics teachers were extracted and summarized into a total of 37 points.

For the dimension of "general teaching", there are 17 items, 10 of which are implemented by teachers and 7 by students. The cognition of pre-service teachers is mainly focused on the fact of "improve the ability to combine numbers and shapes", which was recognized by 19 people, accounting for 37.25% of the total number of them; followed by this is "free use of spatial imagination", which was recognized by 12 people, accounting for 23.53% of the total; next is "hands-on, experience, explore", which was recognized by 11 people, accounting for 21.57% of the total. All of these elements were implemented by students. Thus, for the "general teaching"

dimension, pre-service teachers recognize the importance of students and teachers in implementing intuitive imagination literacy, and they pay more attention to the important role of students as learning agents in this dimension. They realized that to better implement intuitive imagination literacy, students need to have the ability to combine numbers and shapes, spatial imagination, and hands-on practice. Meanwhile, teachers need to have the awareness of developing students' intuitive imagination and have certain teaching skills.

For the dimension of “classroom teaching”, there are 20 items, 11 of which are implemented by teachers and 9 by students. The cognition of pre-service teachers is mainly focused on the fact of “Use visual tools such as graphics”, which was recognized by 31 people, accounting for 60.78% of the total number of them; next is “Demonstrate with the help of modern information technology such as multimedia”, which was recognized by 29 people, accounting for 56.86% of the total. All of these elements were implemented by teachers. Thus, for the “classroom teaching” dimension, the pre-service teachers were also able to recognize the importance of students and teachers in implementing intuitive imagination literacy, and pay more attention to the important role of teachers in classroom teaching. They realize that teachers need to relate to real-life situations in the classroom teaching process and teach with the help of modern information technology and visual tools to stimulate students' interest in learning, and they also emphasize students' direct participation in the learning process. The details are shown in Table 1.

Table 1. Statistics of cognitive focus

Primary Indicators	Secondary Indicators	Code	Content	Number	Percentage
General Teaching	A Teachers	A1	Establish a clear teaching program	5	9.80
		A2	Provide reinforcement to students	2	3.92
		A3	Enhance the focus on core literacy	3	5.88
		A4	Construct a suitable evaluation system	1	1.96
		A5	Cultivate students in a subtle way	1	1.96
		A6	Possess some teaching skills	2	3.92
		A7	Make knowledge visual	8	15.69
		A8	Have the awareness of developing students' intuitive imagination literacy	5	9.80

		A9	Cultivate students' innovative thinking	1	1.96
		A10	Give students the opportunity to solve problems independently on their own	1	1.96
		B1	Learn to paint	1	1.96
		B2	Connect with real life	3	5.88
		B3	Hands-on, experience, explore	11	21.57
	B Students	B4	Free use of spatial imagination	12	23.53
		B5	Develop geometric spatial ability	7	13.73
		B6	Improve the ability to combine numbers and shapes	19	37.25
		B7	Enhance the focus on core literacy	1	1.96
		C1	Demonstrate with the help of modern information technology such as multimedia	29	56.86
		C2	Use visual tools such as graphics	31	60.78
		C3	Combine with life examples	10	19.61
		C4	Stimulate students' interest in learning	6	11.76
	Classroom Teaching	C5	Create appropriate teaching situations	4	7.84
	C Teachers	C6	Demonstrate with the help of physical objects	11	21.57
		C7	Draw pictures normatively	2	3.92
		C8	Strengthen students' thinking training	4	7.84
		C9	Adopt diverse teaching methods	2	3.92
		C10	Implement literacy through geometric proof problems	1	1.96

D Students	C11	Combine specific knowledge to guide students	6	11.76
	D1	Simplify complex problems	1	1.96
	D2	Hands on	7	13.73
	D3	Observe frequently	6	11.76
	D4	Strengthen exercises	6	11.76
	D5	Solve problems with models	11	21.57
	D6	Bold guesses	2	3.92
	D7	Develop good graphing habits	2	3.92
	D8	Master basic graphics	3	5.88
	D9	Understand the conceptual content	1	1.96

4.2 Cognitive Comprehensiveness

This research collated the strategies proposed by previous authors on the development of intuitive imagination literacy and finally summarized 46 points. The details are shown in Table 2.

Table 2 Coding of cultivation strategies proposed by previous authors

Primary Indicators	Secondary Indicators	Code	Content
General Teaching	E Educational Authorities	E1	Balance the development of students' intuitive imagination literacy between geographic regions
		E2	Reinforce the guidance of intuitive imagination strategies and methods
		E3	Focus on the application of modern information technology in developing students' intuitive imagination
	F Teachers	F1	Strengthen the links between disciplines
		F2	Carefully study the curriculum standards, integrate teaching materials, and clarify teaching directions and requirements

	F3	Give recognition and encouragement to students' bold imagination
	F4	Broaden the way, time and space for intuitive imagination teaching
	F5	Innovate teaching methods and enhance feelings of intuition
	F6	Create visual teaching situations
	F7	Guide students to explore in various forms
	F8	Teach students in a subtle way
	F9	Coordinate the development of intuitive imagination and logical reasoning
	F10	Focus on the impact between intuitive imagination literacy and other core mathematical literacies
	F11	Focus on the needs and abilities of students at different levels
	F12	Enhance the development of students' awareness of visual imagination
	G1	Increase awareness of the application of graphics to solve problems
	G2	Form the ability to reason and argue from graphic language to textual language
	G3	Demonstrate, reflect and analyze spatial relationships with the help of nearby objects
G	G4	Enhance communication and imagination
Students	G5	Expand the space for autonomous operation and accumulate geometric intuitive experience
	G6	Seek creative breakthroughs in thinking for analogical learning
	G7	Develop geometric intuition with the help of visual models
	G8	Understand the relationship between vectors and geometry

		G9	Track the dynamics of shapes and explore geometric patterns
		G10	Enhance editing operations to reflect the nature of geometry
		H1	Strengthen practical teaching and focus on visual demonstration teaching
		H2	Encourage students to communicate and express themselves in mathematical language
		H3	Use background materials reasonably
		H4	Make use of typical examples of mathematical culture
		H5	Improve students' interest in learning mathematics
		H6	Strengthen the teaching of parallel and perpendicular relationships in space
		H7	Teach with modern information technology
H	Classroom Teachers	H8	Strengthen the teaching of number and shape combination
		H9	Choose visual tools to provide perceptual understanding
		H10	Cultivate students' habit of reading, drawing and using diagrams
		H11	Focus on the geometric meaning of the explanation, highlighting the essence of the concept
		H12	Guide students to appreciate the mathematical ideas and common mathematical techniques used in the discovery of theorems
		H13	Develop students' ability to draw pictures normatively
I	Students	I1	Focus on mathematical experimental manipulation
		I2	Learn to make connections between graphs and shapes and graphs and quantities

I3	Recognize simple geometry and accumulate representations of spatial figures
I4	Use models of space geometry to solve problems
I5	Learn to reflect spatial figures on a plane
I6	Learn to represent spatial figures with visual diagrams
I7	Focus on variations of visual graphs to deepen understanding of the problem
I8	Use information software to recognize graphical relationships and develop spatial concepts

Matching preservice teachers’ cognition of implementing intuitive imagination literacy with the content of previous studies reveals that current pre-service teachers realize 18 points of the content of previous studies, accounting for 39.13% of the total.

For the dimension of “general teaching”, 25 points were proposed, of which 9 points were recognized by the pre-service teachers, accounting for 36.00%. Among them, pre-service teachers recognized 5 points on the aspect of teachers, accounting for 41.67% of the total number of points in this section, and 4 points on the aspect of students, accounting for 40.00% of the total number of points in this section. However, they have no cognition on the aspect of educational authorities.

For the dimension of “classroom teaching”, 21 points were proposed, of which 9 points were recognized by the pre-service teachers, accounting for 42.86%. Among them, pre-service teachers recognized 6 points on the aspect of teachers, accounting for 46.15% of the total number of points in this section, and 3 points on the aspect of students, accounting for 37.50% of the total number of points in this section.

It can be seen that the current pre-service teachers’ cognition of implementing intuitive imagination literacy is not comprehensive, whether from the overall perspective or two different dimensions of “general teaching” and “classroom teaching”, and it is not half of the number of implementation suggestions put forward by previous authors. In addition, the subjects they recognize for implementation are limited to teachers and students and did not involve the other subjects of implementation proposed by the previous authors. The details are shown in Table 3.

Table 3. Statistics of cognitive comprehensiveness

Primary Indicators	Secondary Indicators	Recognizing Points	Total Points	Percentage	Recognizing Points	Total Points	Percentage
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General Teaching	Educational Authorities	0	3	0.00	9	25	36.00
	Teachers	5	12	41.67			
	Students	4	10	40.00			
Classroom Teaching	Teachers	6	13	46.15	9	21	42.86
	Students	3	8	37.50			
Total Points					18	46	39.13

4.3 Cognitive Reasonableness

The pre-service teachers’ cognition of implementing intuitive imagination literacy was summarized as 37 points, of which 18 points were similar to the suggestions made by the previous authors, accounting for 48.65% of the total. For the dimension of “general teaching”, the pre-service teachers’ statements were summarized as 17 points, of which 9 points were similar to the previous suggestions, accounting for 52.94%; for the dimension of “classroom teaching”, the pre-service teachers’ statements were summarized as 20 points, of which 9 points were similar to the previous suggestions, accounting for 45.00%.

In terms of specific contents, the contents of “use visual tools such as graphics”, “demonstrate with the help of modern information technology such as multimedia” and others, which were recognized by the pre-service teachers, were more consistent with the implementation suggestions made by the previous authors. However, more than half of the contents they recognized, such as “enhance the focus on core literacy” and “improve the ability to combine numbers and shapes”, are not reflected in the implementation suggestions of the previous authors.

It can be seen that the pre-service high school mathematics teachers’ cognition of implementing intuitive imagination literacy was not very reasonable. Less than half of the implementation measures they proposed could match the implementation recommendations made by the previous authors; at the same time, some of the implementation measures that pre-service teachers could realize were unreasonable compared with the previous recommendations and still existed in a certain one-sided way. The details are shown in Table 4 and Table 5.

Table 4. Statistics of cognitive reasonableness

Primary Indicators	Secondary	Recognizing Points	Total Points	Percentage	Recognizing Points	Total Points	Percentage
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Indicators							
General Teaching	Teachers	5	10	50.00	9	17	52.94
	Students	4	7	57.14			
Classroom Teaching	Teachers	6	11	54.55	9	20	45.00
	Students	3	9	33.33			
Total Points					18	37	48.65

Table 5. Statistics of matching situation

Primary Indicators	Secondary Indicators	Code	Content	Number	Percentage
General Teaching	E	E1	Balance the development of students' intuitive imagination literacy between geographic regions	0	0.00
		E2	Reinforce the guidance of intuitive imagination strategies and methods	0	0.00
	Educational Authorities	E3	Focus on the application of modern information technology in developing students' intuitive imagination	0	0.00
		F1	Strengthen the links between disciplines	0	0.00
		F2	Carefully study the curriculum standards, integrate teaching materials, and clarify teaching directions and requirements	5	9.80
F	Teachers	F3	Give recognition and encouragement to students' bold imagination	2	3.92
		F4	Broaden the way, time and space for intuitive imagination teaching	0	0.00
		F5	Innovate teaching methods and enhance feelings of intuition	0	0.00

	F6	Create visual teaching situations	8	15.69
	F7	Guide students to explore in various forms	0	0.00
	F8	Teach students in a subtle way	1	1.96
	F9	Coordinate the development of intuitive imagination and logical reasoning	0	0.00
	F10	Focus on the impact between intuitive imagination literacy and other core mathematical literacies	0	0.00
	F11	Focus on the needs and abilities of students at different levels	0	0.00
	F12	Enhance the development of students' awareness of visual imagination	5	9.80
	G1	Increase awareness of the application of graphics to solve problems	0	0.00
	G2	Form the ability to reason and argue from graphic language to textual language	0	0.00
	G3	Demonstrate, reflect and analyze spatial relationships with the help of nearby objects	3	5.88
G	G4	Enhance communication and imagination	12	23.53
Students	G5	Expand the space for autonomous operation and accumulate geometric intuitive experience	11	21.57
	G6	Seek creative breakthroughs in thinking for analogical learning	0	0.00
	G7	Develop geometric intuition with the help of visual models	7	13.73
	G8	Understand the relationship between vectors and geometry	0	0.00

<p>Classroom Teaching</p> <p>H Teachers</p>	G9	Track the dynamics of shapes and explore geometric patterns	0	0.00
	G10	Enhance editing operations to reflect the nature of geometry	0	0.00
	H1	Strengthen practical teaching and focus on visual demonstration teaching	11	21.57
	H2	Encourage students to communicate and express themselves in mathematical language	0	0.00
	H3	Use background materials reasonably	4	7.84
	H4	Make use of typical examples of mathematical culture	0	0.00
	H5	Improve students' interest in learning mathematics	6	11.76
	H6	Strengthen the teaching of parallel and perpendicular relationships in space	0	0.00
	H7	Teach with modern information technology	29	56.86
	H8	Strengthen the teaching of number and shape combination	0	0.00
	H9	Choose visual tools to provide perceptual understanding	31	60.78
	H10	Cultivate students' habit of reading, drawing and using diagrams	0	0.00
	H11	Focus on the geometric meaning of the explanation, highlighting the essence of the concept	0	0.00
H12	Guide students to appreciate the mathematical ideas and common mathematical techniques used in the discovery of theorems	0	0.00	
H13	Develop students' ability to draw pictures normatively	2	3.92	

	I1	Focus on mathematical experimental manipulation	7	13.73
	I2	Learn to make connections between graphs and shapes and graphs and quantities	0	0.00
	I3	Recognize simple geometry and accumulate representations of spatial figures	3	5.88
I	I4	Use models of space geometry to solve problems	11	21.57
Students	I5	Learn to reflect spatial figures on a plane	0	0.00
	I6	Learn to represent spatial figures with visual diagrams	0	0.00
	I7	Focus on variations of visual graphs to deepen understanding of the problem	0	0.00
	I8	Use information software to recognize graphical relationships and develop spatial concepts	0	0.00

5. DISCUSSION

5.1 Cognitive Focus

According to the above data analysis, it can be seen that for implementing intuitive imagination literacy, current pre-service high school mathematics teachers have recognized two implementation subjects, teachers and students, from different dimensions. For the “general teaching” dimension, pre-service teachers pay more attention to the important role of students as learning agents in this dimension. They realized that to better implement intuitive imagination literacy, students need to have the ability to combine numbers and shapes, spatial imagination, and hands-on practice. Meanwhile, teachers need to have the awareness of developing students’ intuitive imagination and have certain teaching skills. For the “classroom teaching” dimension, the pre-service teachers pay more attention to the important role of teachers in classroom teaching. They realize that teachers need to relate to real-life situations in the classroom teaching process and teach with the help of modern information technology and visual tools to stimulate students’ interest in learning, and they also emphasize students’ direct participation in the learning process. From this we can see that current pre-service high school mathematics teachers can realize the importance of teachers and students for implementing intuitive imagination literacy; students should experience the whole process of intuitive imagination and hands-on practice, while the teacher should play a leading role in the classroom and adopt an effective way of teaching.

5.2 Cognitive Comprehensiveness

According to the above data analysis, it can be seen that the current pre-service teachers' cognition of implementing intuitive imagination literacy is not comprehensive, whether from the overall perspective or two different dimensions of "general teaching" and "classroom teaching", and it is not half of the number of implementation suggestions put forward by previous authors. In addition, the subjects they recognize for implementation are limited to teachers and students and did not involve the other subjects of implementation proposed by the previous authors. From this, we can see that current pre-service high school mathematics teachers do not have a comprehensive cognition of implementing intuitive imagination literacy. This conclusion coincides with the findings of previous studies. Chang believes that at the present stage, front-line teachers' cognition of intuitive imagination often stays at a certain level, such as "shape" supplemented by "number", which neglects the hierarchy of developing intuitive imagination literacy (Chang, 2020).

5.3 Cognitive Reasonableness

According to the above data analysis, it can be seen that the pre-service high school mathematics teachers' cognition of implementing intuitive imagination literacy was not very reasonable. Less than half of the implementation measures they proposed could match the implementation recommendations made by the previous authors; at the same time, some of the implementation measures that pre-service teachers could realize were unreasonable compared with the previous recommendations and still existed in a certain one-sided way. From this, we can see that pre-service high school mathematics teachers do not have a very reasonable cognition for implementing intuitive imagination literacy. Similar findings were found in previous studies. Du found that teachers used many methods to develop students' intuitive imagination literacy in classroom teaching, but they applied them in a non-specific and indiscriminate way, while the effectiveness of some of their suggestions and strategies to develop students' intuitive imagination literacy in mathematics is to be tested (Du, 2019).

6. CONCLUSION AND RECOMMENDATIONS

It has been shown that teachers' cognition for implementing intuitive imagination literacy directly affects the implementation effect of intuitive imagination literacy. The current implementation effect of intuitive imagination literacy in the high school mathematics classroom is not good, so is it that teachers' cognition for implementing intuitive imagination literacy is not comprehensive? Is it not reasonable? In this study, we investigated pre-service high school mathematics teachers' cognitive situation of implementing intuitive imagination literacy. Through investigation and analysis, it can be seen that pre-service high school mathematics teachers do not have a comprehensive cognition for implementing intuitive imagination literacy. Their cognition, both as a whole and in different dimensions, does not reach half of the number of implementation recommendations made by the previous authors, and the subjects they realize for implementation are limited to both teachers and students. In addition, pre-service high school mathematics teachers do not have a very reasonable cognition. Some of their ideas lack feasibility or effectiveness and exist a certain one-sidedness, which needs to be further explored and considered.

According to the above conclusions, it is recommended: 1. The teachers and experts who educate pre-service high school mathematics teachers should pay more attention to intuitive imagination literacy, strengthen pre-service teachers' training in this area, and provide them with more opportunities to practice teaching; 2. Pre-service high school mathematics teachers themselves should take the initiative to study and research, seize the opportunities of teaching exercises, focus on combining theory and practice, and constantly reflect, adjust and improve.

The subjects of this study were 51 masters of education students in the same grade at the same institution, so the limitation of this research is that the sample size was small and did not involve other types of pre-service high school mathematics teacher groups. Therefore, in order to find more detailed and comprehensive results, it is necessary to expand the scope of the research sample and adopt a variety of research methods to conduct a more in-depth study.

Founding

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Competing Interests

The authors declare that they have no competing interests.

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