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THE HISTORY OF MATHEMATICS FOR THE REINFORCEMENT OF STUDENTS' MATHEMATICAL ABILITY AND ATTITUDE

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ABSTRACT
The present research is part of a larger study that aims to investigate whether some mathematical tools, such as spatial ability, geometric transformations, problem posing, history of mathematics etc. can help reduce the students' difficulties which students face in mathematics. In particular, this research, which conducted on students of the first grade of elementary school in Greece for two school years, investigates whether the use of activities based on the History of Mathematics during teaching can improve students' mathematical ability and attitude. The results of this research showed that students gained a positive attitude towards mathematics without any elements of fear and aversion, they recognized their usefulness, and they developed their interest in them, thus enhancing the mathematical perception and mathematical ability of students.

Key Words: History of mathematics, instructional practices, mathematical ability, mathematical attitude.

1. INTRODUCTION
Many international assessment programmes have been developed to assess the effectiveness of the educational systems of the participated countries. PISA (Programme for International Student Assessment), a programme of OECD (Organization for Economic Co-operation and Development), the most widely known assessment programme, is a triennial international survey which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students (PISA, 2016). As the findings are quite reliable, many countries, among which USA, Germany, Finland and Australia, have been led to reform their educational system.

The results of these programmes reveal negative trends of mathematical school education in different educational systems, including the Greek one. Therefore, many researchers worldwide have addressed students' difficulties in mathematics and have proposed methods to diminish them. Specifically, researches findings indicate the critical aspects of the way of teaching mathematics (Lo, 1993; Streefland, 1991; Sfard, 1991; Chen & Li, 2009; Rønning, 2013; Howe et al., 2015; Vlachou & Avgerinos, 2018, Psaras et al., 2020), the use of representations of mathematical concepts (Janvier, 1987; Jiang & Chua, 2010; Shahbari & Peled, 2015; Dreher & Kuntze, 2015; Vlachou & Avgerinos, 2016; Deliyianni et al., 2016; Vlachou & Avgerinos, 2019) and teacher' and prospective teachers' attitudes and beliefs towards mathematics (Lin, 2010;

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Dubinsky, Arnon & Weller, 2013; Tobias, 2013; Avgerinos & Vlachou, 2013; Şahin, Gökkurt & Soylu, 2016; Thanheiser et al., 2016; Whitacre & Nickerson, 2016).

Thus, this paper presents instructional practices based on the history of mathematics and it aims to improve the mathematical perception and ability of students. The didactic interventions were implemented in two school years, 2012-2013 and 2015-2016 in the first grade of the elementary school during the mathematical subject and functioned supportively and complementary to the school books and to curriculum.

2. THEORETICAL FRAMEWORK

History of Mathematics

History of mathematics is used in education, either as a tool or as a goal (Jankvist, 2009). According to Fried (2001), divides the reasons of interest on history of mathematics in relation to mathematics education into three broad themes, that it humanizes mathematics, makes mathematics more interesting, understandable and approachable, and gives insight into concepts, problems and problem-solving.
Tzanakis et al (2002) refer to specific ways of implementing history in mathematics education, including projects based on history texts, historical problems, plays, experiential activities, mistakes, misconceptions, paradoxes of the past.

On the other hand, perceptions play an important role in performance in mathematics, since they affect the way we learn and use mathematics (Goldin, 2003). In addition, students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics (Hart & Walker, 1993). Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, the sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere (Buxton, 1981).

Mathematical Attitude

By the term attitudes we mean the tendencies of the subject to respond in a uniform way to specific events, individuals or lessons. These are permanent tendencies or patterns, emotionally charged reactions of the subject in a situation that involves cognitive and emotional factors. Attitudes contain the element of subjective perception and evaluation of basic parameters of the situation being examined and determine the individual's behavior. The main source of attitudes is previous experiences, positive or negative, that shape a person's emotions. Especially in Mathematics, there are many components of attitudes such as in terms of the difficulty of Mathematics, in terms of their importance or usefulness, and in terms of the enjoyment of dealing with mathematics. Attitudes, in general, seem to play an important role in understanding students' behavior, as positive attitudes are associated with high performance in mathematics (Hart & Walker, 1993; Schoenfeld, 1982).

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In particular, according to Renga & Dalla (1992), teachers can help students gain positive attitudes towards mathematics when they: 1) show enthusiasm and enjoy themselves in mathematics 2) use supervisory tools and active ways to approach mathematical concepts, 3) convincing students about the importance of mathematics, 4) preparing activities in which students can achieve, 5) assessing student attitudes and teaching improvements.

3. METHODS MATERIALS

The research followed a qualitative, quantitative approach. Moreover, a content analysis and case study were carried out. Thus, a triangulation was formed, which was methodological, temporal, topical and theoretical in order to achieve stabilization of findings (Cohen et al. 2011).

The research has been implemented as teaching interventions in two first-grade classes (6-7 years-old pupils) of a Greek primary school, an hours per week for 6 months (for two school years, 2012-2013 and 2015-2016.). The teaching interventions have been designed and implemented by the research team (Psaras et al., 2020).

First grade students have been chosen for the research because at this age they firstly address the formal teaching of mathematics. As a result, the effect of other factors such as previous negative experiences in mathematics, erroneous teaching methods or cognitive deficiencies and misunderstandings are minimal (Psaras, 2020; Avgerinos et al., 2018).

Aim of Research

The main purpose of the present study is to provide instructional practices, which supported in history of mathematics in order to help students develop the mathematical ability and mathematical attitude. That is, students play and learn through original, experiential and modern methods that enhance creativity and critical thinking, love for mathematics with the ultimate goal of developing mathematical ability and student perception.

Participants

The sample of the study involved 29 students of the first grade of elementary school in Greece, age 6-7 years. The sample selection was stratified and symptomatic according to the purposes and needs of the research.

Instrument

Research tools were the activities that were designed based on the history of mathematics. Altogether, 35 activities were designed and implemented, which were applied in addition to 35 of the 63 sections of the mathematics schoolbook of the first grade of the elementary school throughout the school year. In addition, the effect of the interventions was measured by questionnaires on the attitudes of students and their cognitive level. The questionnaire was divided in two parts and was designed by the research team (Psaras et al., 2020; Avgerinos et al., 2018). The first part consisted of 21 clauses in 3-point Likert-type scale regarding the attitudes and beliefs of pupils towards mathematics. There were questions regarding the preference (Bel), the satisfaction (BelEnj), the usefulness (BelUse), the avoidance (BelAvoid) and the motivation

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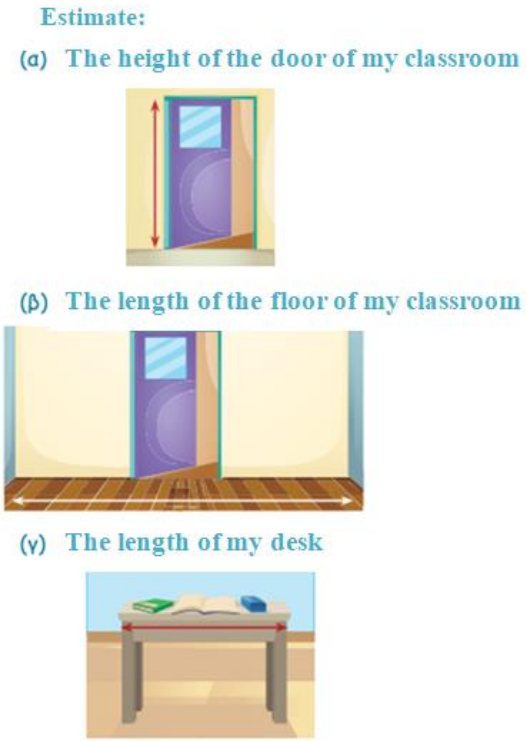
(BelMotiv) related to mathematics learning. The second part of the questionnaire had four tasks, relating to history of mathematics (figure 1 and 2).

Figure 1: The first task of the second part of the questionnaire

Figure 2: The fourth task of the second part of the questionnaire



Data Analysis

In order to analyze the survey data, and in addition to the descriptive analysis, the Statistical Implicative Analysis by Gras (Gras, 1996), using the CHIC (Cohesive Hierarchical Implicative Classification) software (Gras, Suzuki, Guillet, & Spagnolo, 2008) and Microsoft Excel program were used. The implication analysis of data was performed through similarity diagrams, in which

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the variables were associated with each other depending on the similarity or non-similarity they present. Variables in whose solution the subjects behave similarly are grouped together.

Variables of Research

The variables were defined as a combination of letters and one number. The letters indicate the initial of concept which is examined. For example, the variable HiM1 is composed of the initial proposal "History of Mathematics" because the history of mathematics is examined and number 1 indicates the question of questionnaire. According to the implicative analysis, equivalent to a value of 1 was assigned to every item if the answer is correct and 0 if the answer is wrong or missing.

4. TEACHING INTERVENTIONS

The implementation of research was performed through teaching interventions with creative activities based on history of mathematics. 35 activities were designed and implemented, some of which are described below. Many of the activities could be transformed by the teacher and adapted to the specific needs of the class, while a few can be performed in teams.

Indicative Activity 1

Content: History of mathematics and number and operations

Description: In this activity, students try to recognize the numbers of Babylonian numbering system (Figure 3) and they write the corresponding current number (Figure 4).

Figure 3: Image from indicative activity 1

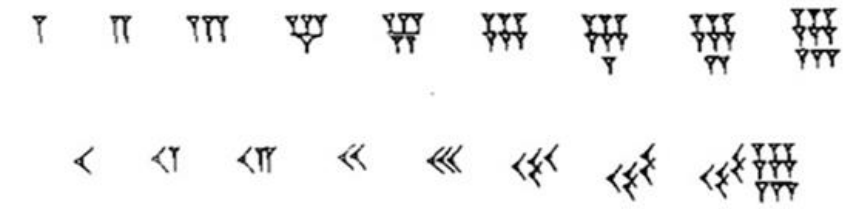

Figure 4: The solution of indicative activity 1

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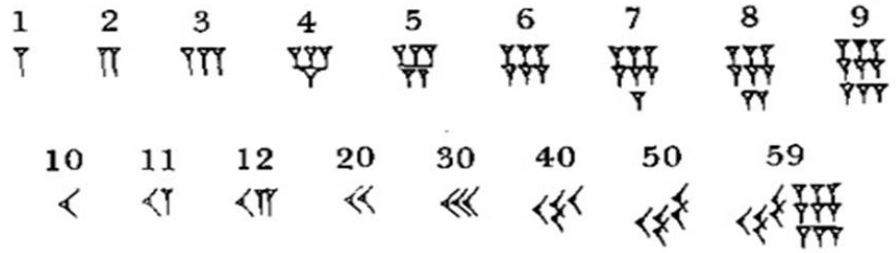

Indicative Activity 2

Content: History of mathematics and geometry

Description: In this activity, it is presented to students the history of "measure", as the communication of peoples and states, with travel and trade, developed, the need arose to establish common units of measurement for better communication. So, in 1791 immediately after the Revolution, the French Academy commissioned a team of scientists from all over Europe to find a simple system of units of measurement. The units that were finally adopted were taken from nature and for the measurement of the length was established the metre which is 1 of the 40,000,000 equal pieces that the earth meridian passing through Paris was divided (Figure 5). Thus, in this activity, students estimate the length of various objects in the classroom.

Figure 5: Units of length - the "metre"

Indicative Activity 3

Content: History of mathematics and geometry

Description: Archimedes' ostomachion is considered the oldest puzzle. It is a square divided into 14 geometric shapes (Figure 6). In the homonymous problem Archimedes (288-212 BC) proves that the ratio of the area of each shape to the original square is an explicit number (all pieces are equal to 1) and seeks the "number" of combinations of its creation. In this activity, some pictures

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of Archimedes' ostomachion are given to the pupils who try to construct them using their own Archimedes' ostomachion (Figure 7).

Figure 6: Archimedes' ostomachion (288-212π.X.)

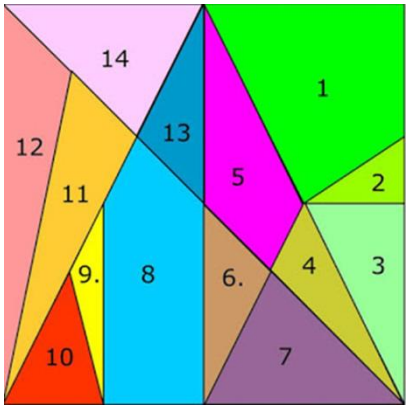
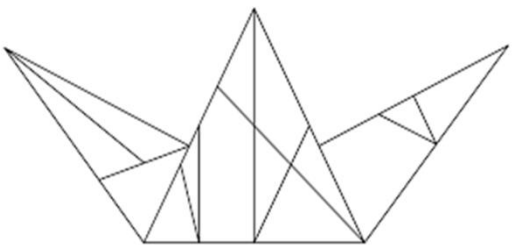
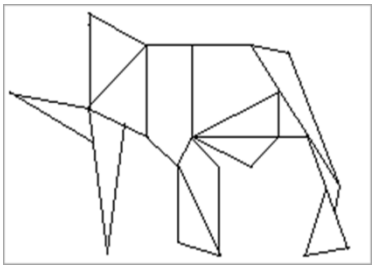


Figure 7: Some pictures of Archimedes' ostomachion





Indicative Activity 4

Content: History of mathematics and geometry

Description: In this activity, it is presented to students the history of tangram and then some pictures of tangram (Figure 7) are given to the pupils who try to construct them using their seven pieces of tangram (Figure 8).

Figure 8: The tangram

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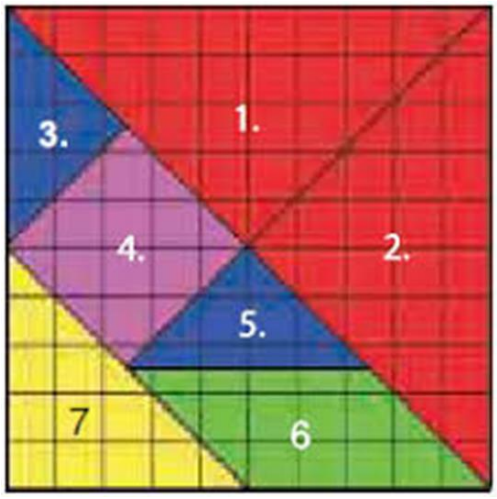

Figure 9: Some pictures of tangram

Indicative Activity 5

Content: History of mathematics and geometry

Description: We are show images to the students from the Greek mosaic works of art of the 5th century BC which represent geometric shapes (Figure 10). Students should analyze the works of art in the individual geometric shapes (Figure 11).

Figure 10: The example of the indicative activity 5

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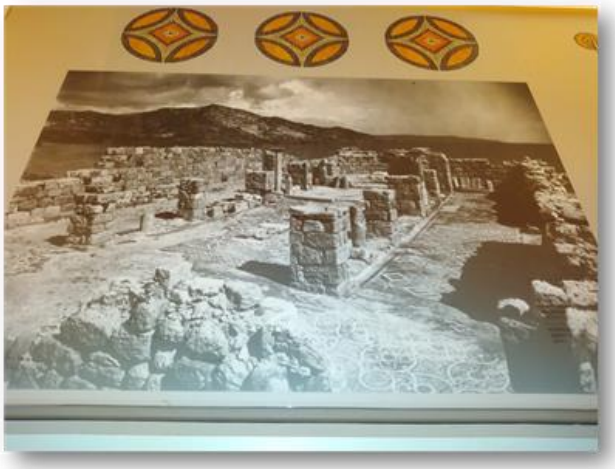

Figure 11: The example of the indicative activity 5

Indicative Activity 6

Content: History of mathematics and number and operations

Description: Sieve of Eratosthenes is an ancient algorithm given by a Greek mathematician named Eratosthenes. It is a simple method for finding prime numbers. In this activity, sieve of Eratosthenes is used by students for learning multiplication of 2, 3 and 5 (Figure 12).

Figure 12: The sieve of Eratosthenes as a key tool for the multiplication of 2, 3 and 5

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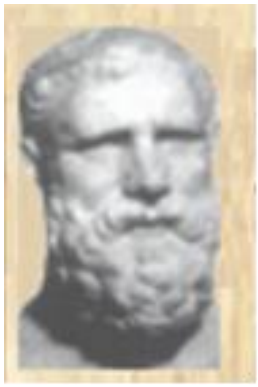

5. RESULTS AND DISCUSSIONS

After implementing the history of mathematics activities, 75% of the students said that their favorite lesson was mathematics. Also, 89% of the students said that they feel pleased when they do mathematics at school (Bel3) and 79% of them feel happy when they read mathematics at home (Bel4). In addition, 91% of students consider mathematics to be a useful subject (BelUse12) and 95% of students said that they like to solve mathematics exercises (Bel16). 90% of students said that they did not scare mathematics (BelFe10) and 95% said that they wanted to be good at mathematics, because they liked it. Also, 95% of the students said that when they did activities on history of mathematics they felt pleasant. Finally, 98% of students responded correctly to the first activity about history of mathematics (HiM1), 85% of students responded correctly to the second activity about history of mathematics and operations (HiMOp2), 91% of students responded correctly to the third activity about history of mathematics and geometry (HiMGe3) and 95% of students responded correctly to the fourth activity about history of mathematics and measure (HiMMe4).

Students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics (Hart & Walker, 1993). Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, Buxton (1981) argues that sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere. In our sample, we observe that 91% of students said that mathematics is a useful lesson, while 90% are not afraid of mathematics, as opposed to research that highlights the aversion and phobia of students about mathematics. In addition, the playful form of activities of history of mathematics has created a positive atmosphere in the classroom and the students' perception that they can do it, as it is a game and not a lesson.

Similarity Analysis

To further understand the connections between history of mathematics and students' mathematical ability and attitude variables, the following similarity analysis was performed. According to similarity tree (Figure 13), in which the variables were associated with each other

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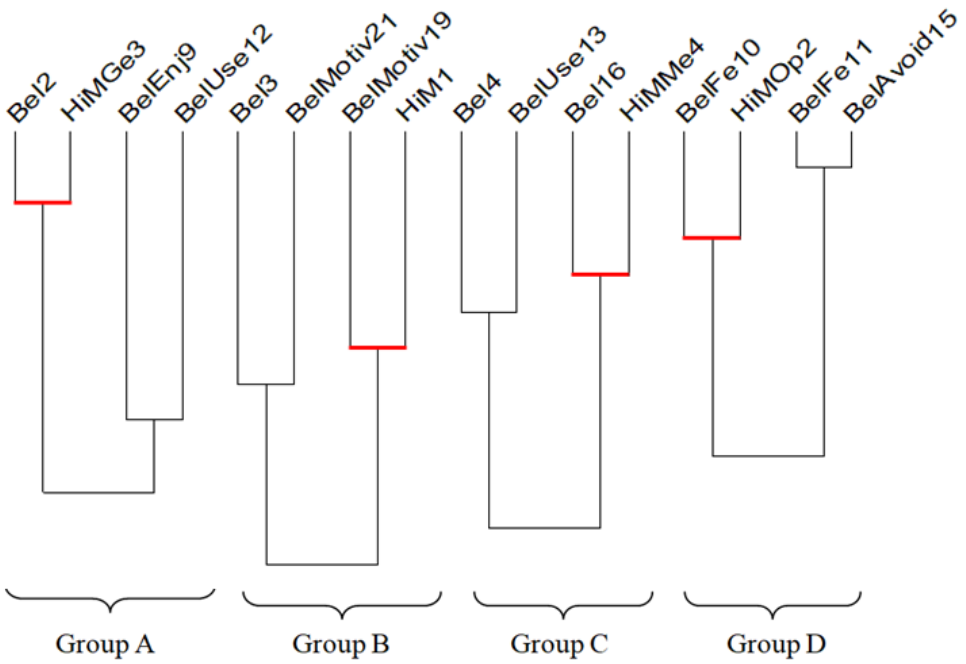
depending on the similarity or non-similarity they present – variables in whose solution the subjects behave similarly are grouped together- it is possible to distinguish observe four groups.

Figure 13: Similarity tree of the solution of the questionnaire



Based on the similarity tree (Figure 13), we observe that a significant relationship has been created in group A between the variables (Bel2 HiMGe3). In particular, the students who correctly solved the activity of geometry with help of history of mathematics (HiMGe3) were the ones who stated that when they read at home the first lesson they choose is mathematics (Bel2). These two variables are related to the variables ((Bel2 HiMGe3) (BelEnj9 BelUse12)) that refer to students who stated that they would like to do more mathematics at school (BelEnj9) and that Mathematics is a useful subject (BelUse12).

Another important relationship created in the similarity tree is that of the group C between the variables (Bel16 HiMMe4). The students, that is, who were able to solve the task, related to the activities about measurement, meter and history of mathematics (HiMMe4), were the ones who stated that they like to solve mathematics exercises (Bel16). These two variables are related to the variable ((Bel4 BelUse13) (Bel16 HiMMe4)). The variable (Bel4) is referred to students who stated that when they do mathematics at home they feel comfortable and the variable (BelUse13) is referred to students who stated that people who do not know mathematics will have difficulty much in their lives.

In addition, another important relationship created in the similarity tree (Figure 13) is that of group B, between variables (BelMotiv19 HiM1). The students who solved correctly the activity of history of mathematics (HiM1) were the ones who stated that they want to be good at

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mathematics to please their parents (BelMotiv19). These two variables (BelMotiv19 HiM1) are related to the variables ((Bel3 BelMotiv21) (BelMotiv19 HiM1)). These variables apply to students who have stated that they feel comfortable doing math at school (Bel3) and that they want to become one of the best students in mathematics (BelMotiv21).

Last but not least, according to similarity tree (Figure 13), in group D, between the variables (BelFe10 HiMOp2) an important relationship has been created. In other words, the students who solution the tasks of operation and history of mathematics, they stated that they were not afraid of mathematics (BelFe10). These two variables (BelFe10 HiMOp2) are related to the variables (BelFe11 BelAvoid15) that refer to students who stated that they do not feel confident about themselves when they do mathematics (BelFe11) and that they avoid studying mathematics (BelAvoid15).

6. CONCLUSION

International assessment programs indicate poor performance in mathematics of students of many countries worldwide with different educational systems, including Greece. In addition, the mathematical phobia is a deterrent to the learning process that negatively affects students' progress. The present research, which is part of a larger study that aims to investigate whether some mathematical tools, such as spatial ability, geometric transformations, problem posing, history of mathematics, realistic mathematics etc. can help reduce the students' difficulties which students face in mathematics, investigates the history of mathematics in the teaching of

elementary school was presented in the course of the mathematics, course based on history of mathematics. These activities, which designed by the researchers themselves (Psaras et al., 2020) were carried out for two school years in elementary school pupils in order to complement the mathematics course and to develop students' mathematical ability and attitude, as both the international literature and international reviews show disappointing evidence about our pupils' performance in mathematics and highlight the phobia and aversion that students feel about them (Hart & Walker, 1993; Mandler, 1989).

In particular, perceptions play an important role in performance in mathematics, since they affect the way we learn and use mathematics (Goldin, 2003). We observe, therefore, that the students who were the sample of our research developed positive attitudes and perceptions regarding the mathematics through the activities of history of mathematics. Thus, the students stated that they like to solve mathematics exercises (95% Bel16), to do more mathematics at school (96% BelEnj9) and that mathematics is a useful subject (91%, BelUse12). In addition, students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics (Hart & Walker, 1993). Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, Buxton (1981) argues that sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere. In our sample, we observe that 91% of students said that mathematics is a useful lesson (BelUse12), while 90% are not afraid of mathematics (BelFe10), as opposed to research

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that highlights the aversion and phobia of students about mathematics. In addition, the playful form of activities about history of mathematcs has created a positive atmosphere in the classroom and the students' perception that they can do it, as it is a game and not a lesson, which agrees with the findings of the international literature according to which the way of teaching affects the performance of students (Lo, 1993; Streefland, 1991; Sfard, 1991; Chen & Li, 2009; Rønning, 2013; Howe et al., 2015).

In conclusion, there were statements about feeling lack of confidence about themselves when students do mathematics (BelFe11) and that they avoid studying mathematics (BelAvoid15). Thus, it is important to reinforce the teaching process of mathematics with various mathematical tools such as the history of mathematics, in order to help our students reduce the difficulties they have in mathematics and to acquire a positive attitude towards them without any elements of fear and aversion thus enhancing mathematical perception and mathematical ability of students.

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