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SPATIAL ABILITY AND GEOMETRIC TRANSFORMATIONS FOR THE REINFORCEMENT OF STUDENTS' MATHEMATICAL ABILITY AND ATTITUDE

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ABSTRACT
According to research's, the mathematical phobia is a deterrent to the learning process that negatively affects students' progress. The present study, based on the above and the poor performance of students in mathematics in several countries, including Greece, in international evaluations such as PISA, investigates whether the use of activities based on Spatial Ability and Geometric Transformations during teaching can improve students' mathematical ability and attitude. The results of this research showed that students gained a positive attitude towards mathematics without any elements of fear and aversion, they recognized their usefulness, and they developed their interest in them, thus enhancing the mathematical perception and mathematical ability of students. Remarkable was the finding where no variable representing the negative attitude of students to mathematics was associated, in the similarity analysis, with variables relating to tasks on spatial ability and geometric transformations.

Key Words: Spatial ability, geometric transformations, mathematical ability, mathematical attitude

1. INTRODUCTION
Many international assessment programmes have been developed to assess the effectiveness of the educational systems of the participated countries. PISA (Programme for International Student Assessment), a programme of OECD (Organization for Economic Co-operation and Development), the most widely known assessment programme, is a triennial international survey which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students (PISA, 2016). As the findings are quite reliable, many countries, among which USA, Germany, Finland and Australia, have been led to reform their educational system.

The results of these programmes reveal negative trends of mathematical school education in different educational systems, including the Greek one. Therefore, many researchers worldwide have addressed students' difficulties in mathematics and have proposed methods to diminish them. Specifically, researches findings indicate the critical aspects of the way of teaching mathematics (Lo, 1993; Streefland, 1991; Sfard, 1991; Chen & Li, 2009; Rønning, 2013; Howe et al., 2015; Vlachou & Avgerinos, 2018, Psaras et al., 2020), the use of representations of

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mathematical concepts (Janvier, 1987; Jiang & Chua, 2010; Shahbari & Peled, 2015; Dreher & Kuntze, 2015; Vlachou & Avgerinos, 2016; Deliyianni et al., 2016; Vlachou & Avgerinos, 2019) and teacher' and prospective teachers' attitudes and beliefs towards mathematics (Lin, 2010; Dubinsky, Arnon & Weller, 2013; Tobias, 2013; Avgerinos & Vlachou, 2013; Şahin, Gökkurt & Soylu, 2016; Thanheiser et al., 2016; Whitacre & Nickerson, 2016).

Thus, this paper presents instructional practices based on the spatial ability and geometric transformations and it aims to improve the mathematical perception and ability of students. The didactic interventions were implemented in two school years, 2012-2013 and 2015-2016 in the first grade of the elementary school during the mathematical subject and functioned supportively and complementary to the school books and to curriculum.

2. THEORETICAL FRAMEWORK

Spatial Ability and Geometric Transformations

Spatial ability is related to human intuition about shapes and their relations. It is generally defined as the ability to generate, retain, retrieve and transform well-structured visual images (Lohman, 1996). It consists of subskills which are related to each other.

The geometry has as its object the study of those properties of the shapes that remain unchanged with respect to all solid motions. By solid motion, we mean the geometric transformations of the plane or space that maintain the distance between two points, the so-called isometries. Thus, we call transformation any unambiguous representation of a set A in a set B, that is, f:A□B (Aglom, 1962). Geometric transformations involve metaphors, displacements, turns, compositions, shape analyzes, reflections, and symmetries.
In the other words, a transformation is a function from a set of points to another, which changes the position (and possibly the size and orientation) of a shape. Spatial ability is a matter of research of many scientific fields and can be developed through training and learning. There is a strong relationship between spatial ability and geometry achievement (Hannafin et al., 2008).

The exercise of children in transformations is considered necessary for the development and conceptual completion of their geometric thinking. Thus, students can improve their visual flexibility, explore properties and relationships, edit shape compositions, make mental transformations, and cultivate the development of pre-conceptual concepts (Avgerinos, Vlachou, Remoundou, 2018).

Mathematical Attitude

By the term attitudes we mean the tendencies of the subject to respond in a uniform way to specific events, individuals or lessons. These are permanent tendencies or patterns, emotionally charged reactions of the subject in a situation that involves cognitive and emotional factors. Attitudes contain the element of subjective perception and evaluation of basic parameters of the situation being examined and determine the individual's behavior. The main source of attitudes is

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previous experiences, positive or negative, that shape a person's emotions. Especially in Mathematics, there are many components of attitudes such as in terms of the difficulty of Mathematics, in terms of their importance or usefulness, and in terms of the enjoyment of dealing with mathematics. Attitudes, in general, seem to play an important role in understanding students' behavior, as positive attitudes are associated with high performance in mathematics (Hart & Walker, 1993; Schoenfeld, 1982).

In particular, according to Renga & Dalla (1992), teachers can help students gain positive attitudes towards mathematics when they: 1) show enthusiasm and enjoy themselves in mathematics 2) use supervisory tools and active ways to approach mathematical concepts, 3) convincing students about the importance of mathematics, 4) preparing activities in which students can achieve, 5) assessing student attitudes and teaching improvements.

3. METHODS MATERIALS

The research followed a qualitative, quantitative approach. Moreover, a content analysis and case study were carried out. Thus, a triangulation was formed, which was methodological, temporal, topical and theoretical in order to achieve stabilization of findings (Cohen et al. 2011).

The research has been implemented as teaching interventions in two first-grade classes (6-7 years-old pupils) of a Greek primary school, an hours per week for 6 months (for two school years, 2012-2013 and 2015-2016.). The teaching interventions have been designed and implemented by the research team (Psaras et al., 2020).

First grade students have been chosen for the research because at this age they firstly address the formal teaching of mathematics. As a result, the effect of other factors such as previous negative experiences in mathematics, erroneous teaching methods or cognitive deficiencies and misunderstandings are minimal.

Aim of Research

The main purpose of the present study is to provide instructional practices, which supported in spatial ability and geometric transformations in order to help students develop the mathematical ability and mathematical attitude. That is, students play and learn through original, experiential and modern methods that enhance creativity and critical thinking, love for mathematics with the ultimate goal of developing mathematical ability and student perception.

Participants

The sample of the study involved 29 students of the first grade of elementary school in Greece, age 6-7 years. The sample selection was stratified and symptomatic according to the purposes and needs of the research.

Instrument

Research tools were the activities that were designed based on the spatial ability and geometric transformations. Altogether, 48 activities were designed and implemented, which were applied in

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addition to 25 of the 63 sections of the mathematics schoolbook of the first grade of the elementary school throughout the school year. In addition, the effect of the interventions was measured by questionnaires on the attitudes of students and their cognitive level. The questionnaire was divided in two parts and was designed by the research team (Psaras et al., 2020). The first part consisted of 20 clauses in 3-point Likert-type scale regarding the attitudes and beliefs of pupils towards mathematics. There were questions regarding the preference (Bel), the satisfaction (BelEnj), the usefulness (BelUse), the avoidance (BelAvoid) and the motivation (BelMotiv) related to mathematics learning. The second part of the questionnaire had four tasks, relating to spatial ability and geometric transformations (figure 1 and 2).

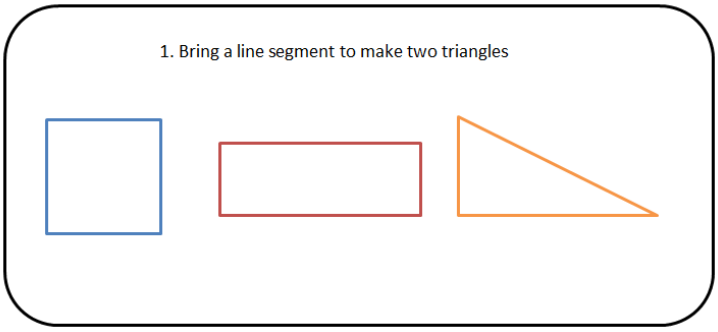

Figure 1: The first task of the second part of the questionnaire

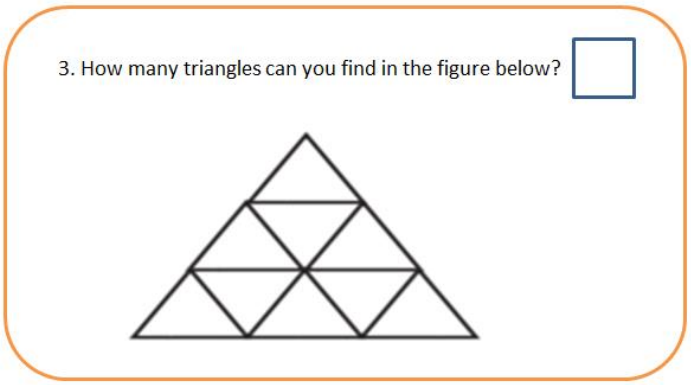

Figure 2: The third task of the second part of the questionnaire

Data Analysis

In order to analyze the survey data, and in addition to the descriptive analysis, the Statistical Implicative Analysis by Gras (Gras, 1996), using the CHIC (Cohesive Hierarchical Implicative Classification) software (Gras, Suzuki, Guillet, & Spagnolo, 2008) and Microsoft Excel program were used. The implication analysis of data was performed through similarity diagrams, in which the variables were associated with each other depending on the similarity or non-similarity they present. Variables in whose solution the subjects behave similarly are grouped together.

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Variables of Research

The variables were defined as a combination of letters and one number. The letters indicate the initial of concept which is examined. For example, the variable GeTrA3 is composed of the initial proposal "Geometric Transformation- Analysis" because the geometric transformation is examined and number 3 indicates the question of questionnaire. According to the implicative analysis, equivalent to a value of 1 was assigned to every item if the answer is correct and 0 if the answer is wrong or missing.

4. TEACHING INTERVENTIONS

The implementation of research was performed through teaching interventions with creative activities based on spatial ability and geometric transformations. 48 activities were designed and implemented, some of which are described below. Many of the activities could be transformed by the teacher and adapted to the specific needs of the class, while a few can be performed in teams. It is noted that some indicative activities are mentioned, as there is an area limitation to show the 48 activities that have been implemented.

Indicative Activity 1

Content: Geometric transformations
Description: Each student has the shape of Figure 3 and tries with the appropriate transformation to arrive at the shape of Figure 4, that is, to form a rectangle. Students experiment with paper shapes (Figure 3) to come up with the right solution.

Figure 3: Original shape

Figure 4: Transformation

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Indicative Activity 2

Content: Spatial ability
Description: In this activity, pupils try to recognize a geometric shape or solid by their hands with closed eyes (Figure 5). By this activity, they analyze the shapes to their properties, which are perceived, by the touch.

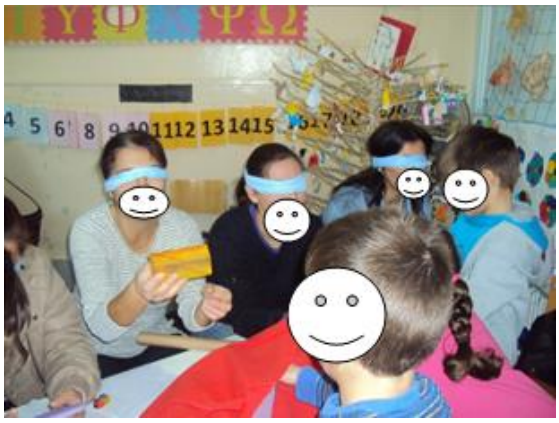

Figure 5: Analyzing the shapes to their properties with tactile contact

Indicative Activity 3

Content: Geometric transformations
Description: In this activity, some pictures of tangram are given to the pupils who try to construct them using their own tangram (Figure 6).

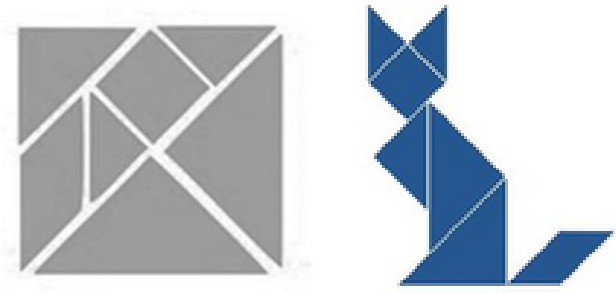

Figure 6: An example -The original shapes are transformed into a cat

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Indicative Activity 4

Content: Geometric transformations
Description: How many combinations can we make with 3 cubes (Figure 7)? Students also process possible transformations in positions, but also relationships of shapes (if the shapes did not have colors, how many are all combinations?).

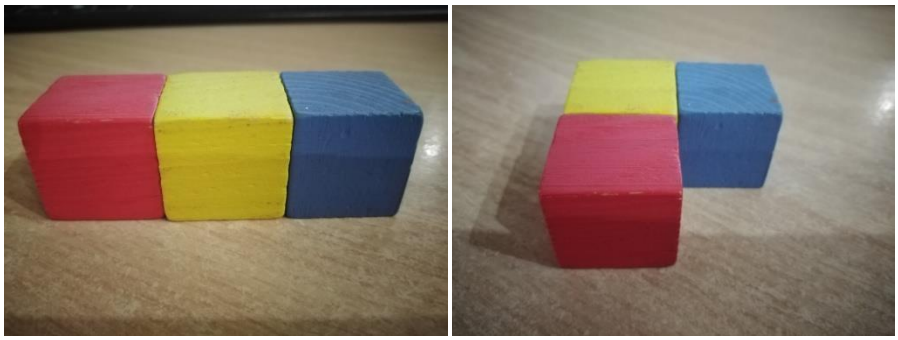

Figure 7: An example of transformation in positions

Indicative Activity 5

Content: Spatial ability
Description: In this activity, students are to imagine the folding and unfolding of pieces of paper. The figures represent a square piece of paper being folded and the last of these figures has one or two small circles drawn on it to show where the paper has been punched. Each hole is punched through all the thicknesses of paper at that point. One of the three figures under the horizontal line shows where the holes will be when the paper is completely unfolded. The students are to decide which one of these figures is correct (Figure 8 and 9). Whenever students are confused by a question, it is helpful to ask them to solve the question using an actual piece of paper themselves.

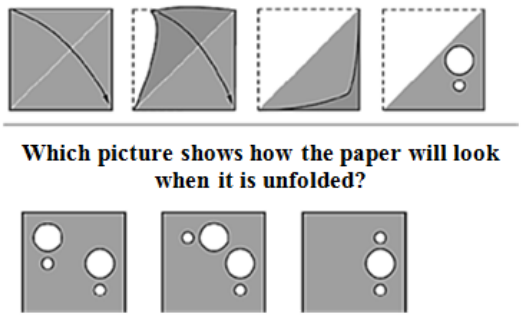

Figure 8: An example of the indicative activity 5

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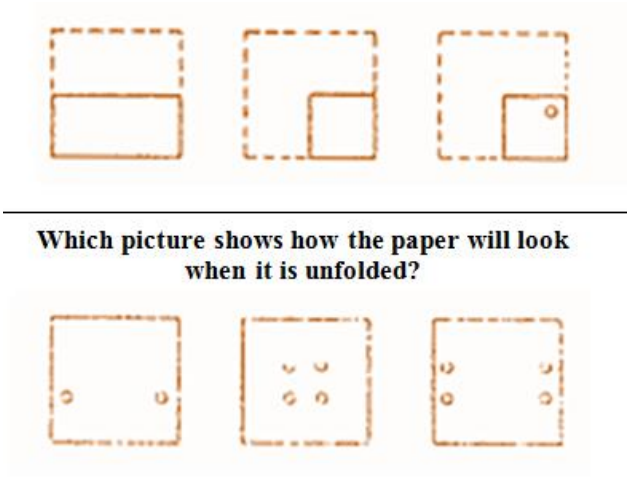

Figure 9: An example of the indicative activity 5

Indicative Activity 6

Content: Geometric transformations
Description: Students are given the 5 red shapes in Figure 10. We ask students how the red shapes move and explain their thinking. Students experiment with wooden shapes to find the solution and explain their answers.

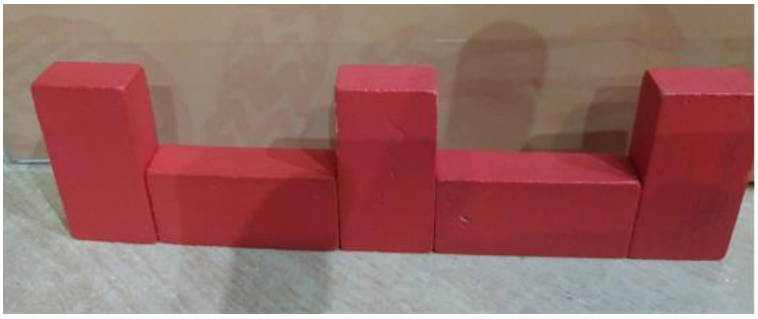

Figure 10: The example of the indicative activity 6

Indicative Activity 7

Content: Geometric transformations
Description: In this activity an approach of geometric transformations through art takes place. More specifically, students are presented different paintings with geometric shapes (Figure 11 and 12), and students will have to find and analyze the paintings in the shapes that make them up.

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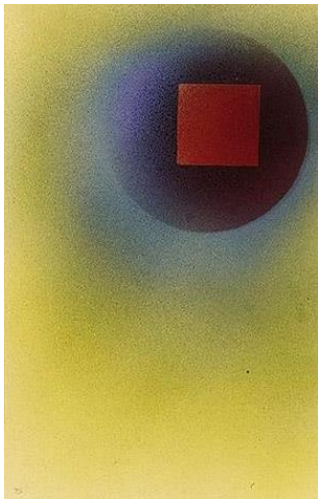

Figure 11: Wassily Kandinsky "Squares ın the circle 1928"

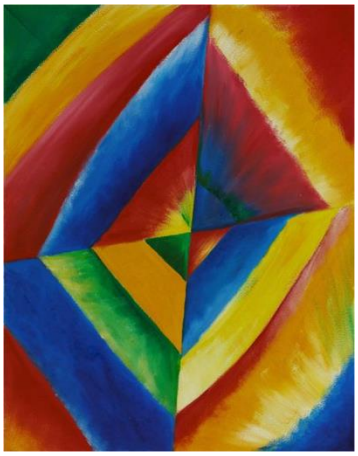

Figure 12: Wassily Kandinsky "Color Studies"

5. RESULTS AND DISCUSSIONS

After implementing the spatial ability and geometric transformations activities, 88% of the students said that their favorite lesson was mathematics. Also, 91% of the students said that they feel pleased when they do mathematics at school and 75% of them feel happy when they read mathematics at home. In addition, 93% of students consider mathematics to be a good lesson, and 97% of students said that they like mathematics. 92% of students said that they did not scare mathematics, and 100% said that they wanted to be good at mathematics, because they liked it. Also, 98% of the students said that when they did the open-ended problems they felt pleasant.

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Finally, 95% of students responded correctly to the first activity about geometric transformation (GeTrS1), 89% of students responded correctly to the second activity about spatial ability (SpAb2), 87% of students responded correctly to the third activity about geometric transformation (GeTrA3) and 90% of students responded correctly to the fourth activity about spatial ability (SpAb4).

Students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics (Hart & Walker, 1993). Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, Buxton (1981) argues that sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere. In our sample, we observe that 93% of students said that mathematics is a useful lesson, while 92% are not afraid of mathematics, as opposed to research that highlights the aversion and phobia of students about mathematics. In addition, the playful form of spatial ability and geometric transformations has created a positive atmosphere in the classroom and the students' perception that they can do it, as it is a game and not a lesson.

Similarity Analysis

To further understand the connections between spatial ability, geometric transformations and students' mathematical ability and attitude variables, the following similarity analysis was performed. According to similarity tree (Figure 13), in which the variables were associated with each other depending on the similarity or non-similarity they present – variables in whose solution the subjects behave similarly are grouped together- it is possible to distinguish observe four groups.

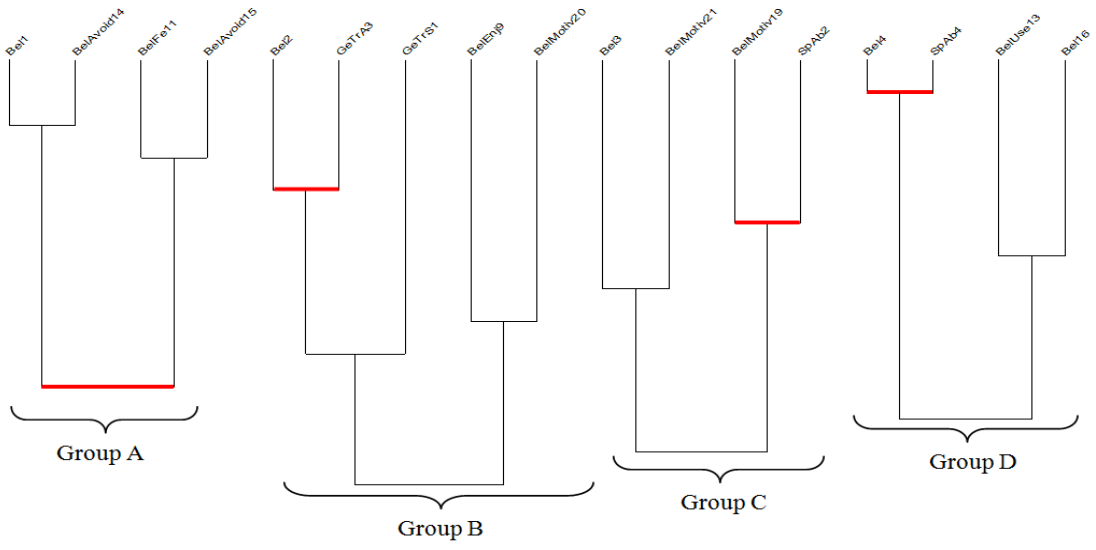

Figure 13: Similarity tree of the solution of the questionnaire

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Based on the similarity tree (Figure 13), we observe that the four similarity groups have been formed. We notice that a significant relationship has been created in group D between the variables (Bel4 SpAb4). In other words, the students who correctly solved the activity of spatial ability (SpAb4) were the ones who stated that they like to solve mathematics exercises at home (Bel4). These two variables are related to the variables ((Bel4 SpAb4) (BelUse13 Bel16)) that refers to students who stated that they like to solve mathematics exercises (Bel16) and they stated that people who do not know mathematics will have difficulty much in their lives (BelUse13).

Another important relationship created in the similarity tree is that of the group B between the variables (Bel2 GeTrA3). The students, that is, who were able to solve the task related to the analysis of geometric transformations, were the ones who stated that when they read at home the first lesson they choose is mathematics. These two variables (Bel2 GeTrA3) are related to the variable ((Bel2 GeTrA3) GeTrS1). This variable (GeTrS1) refers to the task involved in the synthesis of geometric transformations. That is, we observe a similar way of solving the exercises with the geometric transformations by the students which related to the variables (((Bel2 GeTrA3) GeTrS1) (BelEnj9 BelMotiv20)). In other words, the students who solution the tasks of the geometric transformations, they stated that would like to do more mathematics at school (BelEnj9) and that they want to be good at mathematics, because they like it (BelMotiv20).

In addition, another important relationship created in the similarity tree is that of group C, between variables (BelMotiv19 SpAb2). The students who did not solve correctly the activity of spatial ability (SpAb2) were the ones who stated that they want to be good at mathematics to please their parents. These two variables (BelMotiv19 SpAb2) are related to the variables ((Bel3 BelMotiv21) (BelMotiv19 SpAb2)). These variables apply to students who have stated that they feel comfortable doing math at school (Bel3) and that they want to become one of the best students in mathematics (BelMotiv21).
Finally, in group A, between the variables ((Bel1 BelAvoid14) (BelFe11 BelAvoid15)) an important relationship has been created. In particular, we have variables that express students' statements that mathematics is not their favorite subject (Bel1), that they study mathematics because their parents force them to (BelAvoid14), that they do not feel confident about themselves when they do mathematics (BelFe11), and finally that they avoid studying mathematics (BelAvoid15). Notable in this group A of variables relating to mathematical phobia is that they are not related to any variable relating to spatial ability and geometric transformations.

6. CONCLUSION, DISCUSSIONS, AND RECOMMENDATIONS

International assessment programs indicate poor performance in mathematics of students of many countries worldwide with different educational systems, including Greece. In addition, the mathematical phobia is a deterrent to the learning process that negatively affects students'

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progress. In the present research, based on the above, a part of the activities implemented in the 1st grade of the elementary school was presented in the course of the mathematics, course based on spatial ability and geometric transformations. These activities, which designed by the researchers themselves (Psaras et al., 2020) were carried out for two school years in elementary school pupils in order to complement the mathematics course and to develop students' mathematical ability and attitude, as both the international literature and international reviews show disappointing evidence about our pupils' performance in mathematics and highlight the phobia and aversion that students feel about them (Hart & Walker, 1993; Mandler, 1989).

In particular, perceptions play an important role in performance in mathematics, since they affect the way we learn and use mathematics (Goldin, 2003). We observe, therefore, that the students who were the sample of our research had positive attitudes and perceptions regarding the mathematics developed through the activities of spatial ability and geometric transformations. In addition, students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics (Hart & Walker, 1993). Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, Buxton (1981) argues that sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere. In our sample, we observe that 93% of students said that mathematics is a useful lesson, while 92% are not afraid of mathematics, as opposed to research that highlights the aversion and phobia of students about mathematics. In addition, the playful form of spatial ability and geometric transformations has created a positive atmosphere in the classroom and the students' perception that they can do it, as it is a game and not a lesson, which agrees with the findings of the international literature according to which the way of teaching affects the performance of students (Lo, 1993; Streefland, 1991; Sfard, 1991; Chen & Li, 2009; Rønning, 2013; Howe et al., 2015).

In conclusion, the results of this research showed that students who participated in the activities of spatial ability and geometric transformations gained a positive attitude towards mathematics without any elements of fear and aversion, they recognized their usefulness, they developed their interest in them, thus enhancing the mathematical perception and mathematical ability of students. Remarkable was the finding, which confirms the above, where no variable representing the negative attitude of students to mathematics was associated, in the similarity analysis, with variables relating to tasks on spatial ability and geometric transformations.

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